The Fourth Dimension and Non-Euclidean Geometry in Modern Art

Introduction

During the years 1902 to 1907, at the time Albert Einstein was working in the Swiss Patent Office, Charles Howard Hinton, a little-known Englishman crucial to this study, was employed in the United States Patent Office in Washington, D.C. Hinton had published his last major book, *The Fourth Dimension*, in 1904, three years before his death at age fifty-four. Einstein, on the other hand, was at the threshold of his own life's work, in 1905 formulating the first of his many contributions to science, the Special Theory of Relativity. In the long run, Einstein's influence was to be far greater than that of Hinton, revolutionizing scientific theory and, after about 1919, the world view of laymen as well. However, in the first two decades of the twentieth century, the idea promulgated by Hinton and many others that space might possess a higher, unseen fourth dimension was the dominant intellectual influence.

The complex spatial possibilities suggested by a fourth dimension, as well as by the curved space of non-Euclidean geometry, were the outgrowth of developments in early nineteenth-century geometry. Popularized during the later years of the century, these notions had begun to capture the public's imagination by the turn of the century in much the same way Black Holes have done in recent years. Like a Black Hole, "the fourth dimension" possessed mysterious qualities that could not be completely understood, even by scientists themselves. Yet, the impact of "the fourth dimension" was far more comprehensive than that of Black Holes or any other more recent scientific hypothesis except Relativity Theory after 1919. Emerging in an era of dissatisfaction with materialism and positivism, "the fourth dimension" gave rise to entire idealist and even mystical philosophical systems, such as that of Hinton. Only the popularization of Einstein's General Theory of Relativity, with its redefinition of the fourth dimension as time instead of space,[1] brought an end to this era in which artists, writers, and musicians believed they could express higher spatial dimensions.

Besides the artists who are the subject of this study, the list of prominent figures interested in the fourth dimension is an impressive one. Between Dostoevsky's references to higher dimensions and non-Euclidean geometry in *The Brothers Karamazov* of 1880 and P. G. Wodehouse's offhanded use of the term in "The Amazing Hat Mystery" of 1922, the fourth dimension attracted the notice of such literary figures as H. G. Wells, Oscar Wilde, Joseph Conrad, Ford Madox Ford, Marcel Proust, and Gertrude Stein.[2] Among musicians, Alexander Scriabin, Edgar Varèse, and George

[1] See Appendix A for the principles of Relativity Theory in the various phases of its development and the timing of its popularization.

[2] Except for Dostoevsky and Wodehouse, all of these

Antheil were actively concerned with the fourth dimension, and were encouraged to make bold innovations in the name of a higher reality.

For early twentieth-century artists "the fourth dimension" and non-Euclidean geometry had an equally liberating effect. In the past, art historians have most often ignored or dismissed references to either of the "new geometries" in the writings of modern artists and critics.[3] Without a knowledge of the widespread popular interest in these spatial concepts, historians tended to misinterpret the terms as purely mathematical or purely mystical, missing the variety of views between the two extremes. When understood in their original context, however, "the fourth dimension" and non-Euclidean geometry are far from being the "scourge of every history of modern painting,"[4] as they have been termed. Instead, these concepts open the door to our understanding more fully the goals of many seminal artists of the early twentieth century.

That references in Cubist literature to higher dimensions and to non-Euclidean geometry had nothing to do with Einsteinian Relativity Theory was first suggested by John Adkins Richardson and myself in the early 1970s. My historical arguments against such a connection, presented in a 1971 *Art Quarterly* article,[5] are in Appendix A below, which includes additional information on the subject as well. In his 1971 text *Modern Art and Scientific Thought* Richardson supported his case by citing the works of J.C.F. Zöllner and H. G. Wells as proof that the term *the fourth dimension* was used independently of Relativity Theory in this period.[6] Although my *Art Quarterly* article

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authors are discussed in the pages that follow. Dostoevsky's Ivan Karamazov refers to higher dimensions and non-Euclidean geometry in the course of his speculation on the existence of God (Fyodor Dostoyevsky, *The Brothers Karamazov*, trans. Constance Garnett [New York: New American Library, 1957], pp. 216-17). For Wodehouse's tale, see P. G. Wodehouse, *Young Men in Spats* (Harmondsworth, England: Penguin Books, 1971), pp. 68-86. The magical properties of higher spatial dimensions, which fascinated H. G. Wells and, before him, Lewis Carroll, have continued to stimulate writers of science fiction. See, for example, Robert A. Heinlein, " '—And He Built a Crooked House—,' " *Astounding Science Fiction*, February 1941; reprinted in *Analog's Golden Anniversary Anthology*, ed. Stanley Schmidt (New York; Davis Publications, 1980), pp. 95-110. For an overview of this body of literature, which continued to grow even after the popularization of Relativity Theory, see Pierre Versins, *Encyclopédie de l'utopie, des voyages extraordinaires, et de science-fiction* (Lausanne: L'Age d'Homme, 1972).

[3] "New geometry" is used throughout this study as a relative term, in that the *n*-dimensional and non-Euclidean geometries, which seemed so novel and modern at the turn of the century, had actually existed since the first half of the nineteenth century.

[4] William Rubin, "Reflexions on Marcel Duchamp," *Art International*, IV (1 Dec. 1960), 52. A recent sign of how the new geometries are increasingly recognized for their role in the evolution of modern art is the essay by Lucy Adelman and Michael Compton, "Mathematics in Early Abstract Art," in *Towards a New Art: Essays on the Background to Abstract Art*, ed. Michael Compton (London: The Tate Gallery, 1980), pp. 64-89.

[5] Henderson, "A New Facet of Cubism: 'The Fourth Dimension' and 'Non-Euclidean Geometry' Reinterpreted," *The Art Quarterly*, XXXIV (Winter 1971), 410-33.

[6] See Richardson, *Modern Art and Scientific Thought* (Urbana: University of Illinois Press, 1971), ch. 5, "Cubism and Logic." His chapter had been published

presented the first extended discussion of the popular tradition of "the fourth dimension" and non-Euclidean geometry as an outgrowth of nineteenth-century geometry, several other art historians, in addition to Richardson, had earlier touched upon one or another of its manifestations. Christopher Gray in *Cubist Aesthetic Theories* of 1953 had noted the mystical writings of P. D. Ouspensky on the subject.[7] Similarly, Apollinaire scholars LeRoy C. Breunig and Jean-Claude Chevalier in 1965 had mentioned the 1909 *Scientific American* essay contest on the fourth dimension, as well as the 1912 *Comoedia* serial *Voyage au pays de la quatrième dimension*.[8] Nevertheless, only one historian of modern art, Meyer Schapiro, seems to have sensed the broader philosophical implications of the new geometries in the early twentieth century. As early as his 1937 essay on the "Nature of Abstract Art" Schapiro noted, "Just as the discovery of non-Euclidean geometry gave a powerful impetus to the view that mathematics was independent of existence, so abstract painting cut at the roots of the classic ideas of artistic imitation."[9]

My reinterpretation of Cubist references to a fourth dimension and to non-Euclidean geometry was expanded in a 1975 Ph.D. dissertation to include Marcel Duchamp, as well as a number of artists working outside of France during the period 1900 to 1930.[10] The present text is a more fully developed version of that work, with a new chapter added on American art. France, however, remains the central focus of this study as well, for it was among the Cubists that the first and most coherent art theory based on the new geometries was developed. From Cubism (and the speculation of Duchamp) successive artistic explorations of the subject occurred in Italy, America, Russia and Holland. The varying approaches of artists in each of these nations toward the new geometries and their frequent alterations of the original prewar Cubist ideas on the subject provide valuable new insights into the character of a number of modern movements.

As wide ranging as the present study is, the absence of two avant-garde centers, England and Germany, may initially raise questions. Yet this omission was dictated by my requirement that for each artist or movement to be examined there be a body of writings on the fourth dimension and non-Euclidean geometry by an artist and his contemporaries. Thus, while the fourth dimension was certainly well known in England

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in much the same form in France in 1969 as "Un Mythe de la critique moderne: Le Cubisme et la quatrième dimension," *Diogène*, no. 65 (Jan.–Mar. 1969), pp. 103-15.

[7] See Gray, *Cubist Aesthetic Theories* (Baltimore: Johns Hopkins Press, 1953), p. 85 and n. 74 to p. 85.

[8] Breunig and Chevalier, *Guillaume Apollinaire: Les Peintres Cubistes* (Paris: Hermann, 1965), p. 105.

[9] Schapiro, "Nature of Abstract Art," *Marxist Quarterly*, I (Jan.–Mar. 1937), 78.

[10] Henderson, "The Artist, 'The Fourth Dimension,' and Non-Euclidean Geometry 1900-1930: A Romance of Many Dimensions," Ph.D. dissertation, Yale University, 1975.

and Germany, it does not figure prominently in Vorticist or German Expressionist literature.[11]

Germany, and particularly the Russian-born leader of the Munich Blaue Reiter group, Wassily Kandinsky, proved the most surprising in this regard. As is discussed in Chapter 5, Kandinsky was clearly aware of the fourth dimension and shared an anti-materialist stance with his Russian colleagues that would have made the fourth dimension a logical part of his artistic theory. The term, however, does not appear in Kandinsky's published writings, which instead bear the stamp of Rudolf Steiner's Christian Theosophy. In fact, in Germany it may have been the strong influence of Steiner that overwhelmed "the fourth dimension" with a compatible but more mystical and elaborate philosophy.[12]

Among German Expressionist painters there may also have been a conscious rejection of a fourth dimension identified with cerebral French Cubism. Significantly, the Cubist painter who had the most widespread impact on Germany, Robert Delaunay, was not a major advocate of the fourth dimension.[13] Thus, Franz Marc, cofounder with Kandinsky of the Blaue Reiter group and an admirer of Delaunay, documented his own interest in the fourth dimension only in 1916. In a letter to his wife of 24 January 1916, Marc wrote enthusiastically of hearing from a physicist friend about the progress of science beyond three dimensions to four-dimensional space-time.[14]

The question of the artistic influence of the fourth dimension in Germany, nevertheless, deserves further research. Germany, after all, had produced more scholarly and semipopular articles on this subject than any other country by 1910.[15] This literature needs to be surveyed carefully, and the writings of German artists, as well

[11] As will be seen, the notion of a vortex itself had been connected to the fourth dimension by Hinton. However, although there was a strongly geometrical orientation in Vorticist art (advocated particularly by T. E. Hulme) and Ezra Pound was later vocal in his support of George Antheil's musical interest in a fourth dimension, Wyndham Lewis's desire to prove Vorticism independent of its Cubist and Futurist sources may have caused him to downplay the fourth dimension. Only later did Lewis write on the subject—after its redefinition by Einstein—in *Time and Western Man* (New York: Harcourt, Brace & Co., 1928).

[12] Although the fourth dimension never became a vital part of his Theosophical system, Steiner had actually lectured on the subject in 1904-1905 (Robert C. Williams, *Artists in Revolution: Portraits of the Russian Avant-Garde 1905-1925* [Bloomington: Indiana University Press, 1977], p. 109).

[13] By the time Apollinaire traveled to Berlin with Delaunay in early 1913, he was no longer so interested in the fourth dimension. Nevertheless, the German avant-garde would have learned of his earlier views on Cubism and the fourth dimension in *Les Peintres Cubistes*. For example, Paul Fechter in his 1914 text *Der Expressionismus* cynically chided Apollinaire for neglecting in *Les Peintres Cubistes* to discuss Riemann and non-Euclidean geometry in connection with the fourth dimension (Paul Fechter, *Der Expressionismus* [Munich: R. Piper & Co., 1914], p. 34).

[14] Franz Marc letter to Maria Marc, 24 January 1916, in Marc, *Briefe, Aufzeichnungen und Aphorismen* (Berlin: Paul Cassirer, 1920), p. 104. Marc's change in attitude toward science is discussed by Ida Katherine Rigby in "Franz Marc's Wartime Letters from the Front," in *Franz Marc, 1880-1916*, ex. cat. (University Art Museum, University of California, Berkeley, 5 Dec. 1979–3 Feb. 1980), p. 58.

[15] See Duncan M. Y. Sommerville, *Bibliography of*

as figures such as Kandinsky's friend Arnold Schoenberg, should be examined closely for evidence of the influence of the new geometries, even in their most subtle, non-mathematical forms.[16] For instance, Schoenberg's advocacy of a new atonal language for music reflects one of the most persistent themes associated with "the fourth dimension": the inadequacy of present language to deal with the new reality of higher dimensions.

The present study, however, concentrates on the early twentieth-century artists and movements that have left the most direct records of their interest in the spatial concepts associated with the new geometries. Even in the absence of advocates among the German Expressionists and English Vorticists, the fourth dimension and non-Euclidean geometry emerge as among the most important themes unifying much of modern art and theory.

Non-Euclidean Geometry, Including the Theory of Parallels, the Foundations of Geometry, and Space of n *Dimensions* (London: Harrison & Sons, 1911), p. viii.

[16] Another figure for investigation is Herwarth Walden, founder of the Berlin gallery and periodical *Der Sturm*. Walden must have had at least a passing interest in the fourth dimension, for he published in 1911 a review by S. Friedlaender-Halensee of Max Zerbst's *Die vierte Dimension* (*Der Sturm*, II [Oct. 1911], 663-64).

The Fourth Dimension and Non-Euclidean Geometry in Modern Art

1

The Nineteenth-Century Background

The two geometries that were to fascinate artists in the first decades of the twentieth century had been developed more than sixty years earlier. The principles of a consistent non-Euclidean geometry were initially formulated in the 1820s, while the first major discussions of *n*-dimensional geometry were published in the 1840s.

Non-Euclidean Geometry

Non-Euclidean geometry was named for its opposition to one of the postulates Euclid had set forth as the basis of his deductive system of geometry in the *Elements* (ca. 300 B.C.). Even the earliest commentators on Euclid felt that his fifth postulate was not as self-evident as the others and must somehow be deducible from another of his postulates or common notions. The famous "parallel postulate," as it is called, states, "That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the straight lines, if produced indefinitely, meet on that side on which are the angles less than two right angles."[1] More familiar is the equivalent formulation of this notion: "Through a given point can be drawn only one parallel to a given line."[2]

Over the centuries all attempts at a direct and, later, an indirect proof of the parallel

[1] *The Thirteen Books of Euclid's Elements*, trans. Thomas L. Heath, 3 vols., 2nd ed. (Cambridge: The University Press, 1926), I, p. 155.

[2] Harold E. Wolfe, *Introduction to Non-Euclidean Geometry* (New York: Holt, Rinehart & Winston, 1945), p. 20.

postulate were unsuccessful.[3] By 1824 Karl Friedrich Gauss had concluded that alternative geometries to Euclid's must be possible. Gauss never published his thoughts on non-Euclidean geometry, and so the honor of its official discovery has been given to Nikolai Ivanovich Lobachevsky, a Russian, and Janos Bolyai, a Hungarian, who separately formulated the first system of non-Euclidean geometry. In 1829 Lobachevsky published "On the Principles of Geometry" in the *Kazan Messenger*, describing the "imaginary geometry" he had developed as early as 1826.[4] Bolyai's work on the "Absolute Science of Space," as he called it, appeared only in 1832 as an appendix to his father's mathematical treatise, *Tentamen*, although he had completed his manuscript by 1829.[5]

Lobachevsky and Bolyai chose the same alternative to the parallel postulate: Through a given point not on a given line, more than one line can be drawn not intersecting the given line. In such a geometry, then, an infinite number of lines can be drawn through a point, and, although these lines may approach a given line as they are extended to infinity, they will never intersect it. Similarly, the sum of the angles of a triangle will be less than the familiar 180° of Euclidean geometry. The visualization of these properties of figures in the Lobachevsky-Bolyai geometry was greatly facilitated in 1868 when the Italian mathematician Beltrami proposed the "pseudosphere" as a partial model for this type of non-Euclidean geometry (Fig. 1). On such a surface of constant negative curvature one can more readily imagine how a group of lines might be parallel to another, approaching it but never intersecting it, and how the angle sum of a triangle can be less than 180°.

Beltrami's introduction of the pseudosphere in 1868 indicated a sudden sharpening of interest among mathematicians in non-Euclidean geometry, after several decades in which the research of Lobachevsky and Bolyai remained practically unknown. Bolyai had in fact published little beyond his appendix for the *Tentamen*. Lobachevsky, on the other hand, had continued to write on his new geometrical discoveries, but these works were primarly in Russian.[6] The situation changed radically during the 1860s in

[3] See Roberto Bonola, *Non-Euclidean Geometry: A Critical and Historical Study of Its Development*, trans. H. S. Carslaw (Chicago: Open Court, 1912) for an account of the attempts of Gerolamo Saccheri and others to prove the parallel postulate.

[4] On Gauss and Lobachevsky, see Carl B. Boyer, *A History of Mathematics* (New York: John Wiley & Sons, 1968), pp. 567, 586-87. For Lobachevsky's career and geometrical philosophy, see Alexander Vucinich, "Nikolai Ivanovich Lobachevskii: The Man Behind the First Non-Euclidean Geometry," *Isis*, LIII (Dec. 1962), 465-81. See also Chapter 5, n. 14, below.

[5] Wolfe, *Introduction*, p. 52.

[6] The major exceptions in Lobachevsky's predominantly Russian bibliography were the article "Géométrie imaginaire," published in French in a German journal in 1837, and the 1840 treatise *Geometrische Untersuchungen zur Theorie der Parallellinien*, published in Berlin. See the Bibliography, sec. I, A, for a listing of Lobachevsky's and Bolyai's major works and translations of these. An indispensable aid in dealing with the history of non-Euclidean (and *n*-dimensional) geometry is Sommerville, *Bibliography of Non-Euclidean Geometry* (Intro., n. 15, above).

France, however, when Hoüel translated into French an important treatise by each of these men, adding extracts of Gauss's correspondence on non-Euclidean geometry to the Lobachevsky translation.[7] The name of Gauss gave new prestige to non-Euclidean geometry, which now captured the interest of a younger generation of mathematicians, who developed it further.

The year 1867 witnessed the publication of Georg Friedrich Bernhard Riemann's now famous speech of 1854, in which the idea for the other major type of non-Euclidean geometry was suggested. Riemann's lecture before the faculty of the University of Göttingen, "Über die Hypothesen, welche der Geometrie zu Grunde liegen," offered a broad new view of geometry, as well as specific suggestions for another alternative to Euclid's system. Riemann saw geometry in general as the study of manifolds of any number of dimensions and of any curvature, using differential geometry as the measure of this curvature.

In the course of his discussion of this new approach to geometric space, Riemann pointed out for the first time the important distinction between unbounded space and infinite space. On the surface of a sphere space would be unbounded and yet finite, and the sphere, in fact, is the most easily understood model for the non-Euclidean geometry implied by Riemann. Once space is finite and a line cannot be extended indefinitely (as Euclid's parallel postulate assumes it will be), it is possible to establish that *no* line can be drawn parallel to a given line.[8] This principle is readily apparent in the geometry of the sphere where "lines" are defined as great circles and will all intersect at the "poles" of the sphere (Fig. 2). From the analogy with spherical geometry, it is also clear that the sum of the angles of a triangle will be greater than 180°. Riemann's geometry on surfaces of constant positive curvature is thus the opposite of the Lobachevsky-Bolyai geometry of surfaces of constant negative curvature.

Riemann's metrical approach to geometry and his interest in the problem of congruence also gave rise to another type of non-Euclidean geometry, in this case defined not by its rejection of the parallel postulate but rather by its irregular curvature. While in a system of Lobachevskian or Riemannian geometry based on alternatives to the parallel postulate the measure of curvature must be constant, Riemann's broad view of geometry had suggested the possibility of surfaces or spaces where curvature might vary. On such an irregularly shaped surface, a figure could not be moved about without changes occurring in its own shape and properties.[9] Although Euclid had not formally

[7] Bonola, *Non-Euclidean Geometry*, p. 123. See also Sommerville, *Bibliography*, p. v.

[8] Wolfe, *Introduction*, p. 61.

[9] On non-Euclidean geometry considered in terms of congruence, see A. d'Abro, *The Evolution of Scientific Thought from Newton to Einstein* (New York: Boni & Liveright, 1927; 2nd ed., rev. and enl. New York: Dover, 1950), ch. 3, "Riemann's Discoveries and Congruence." For the work of Helmholtz and Lie on this subject, see Bonola, *Non-Euclidean Geometry*, pp. 152-54.

The remaining history of non-Euclidean geometry postulated the indeformability of figures in movement, this assumption is essential to his system.[10] When the principle of indeformability is negated, a geometry results in which figures may "squirm" when they are moved about.

It was this latter type of non-Euclidean geometry that would be of greatest interest to artists of the early twentieth century, such as the Cubists and Marcel Duchamp. In the end, too, the primary non-Euclidean characteristic of Einstein's space-time continuum would be its variable curvature from place to place, caused by the gravitational force of the matter distributed throughout the continuum. Yet the two types of non-Euclidean geometry (deriving either from alternatives to the parallel postulate or from questions of congruence) shared a critical and provocative idea: the possibility of curved space. The suggestions that space beyond our immediate perceptions might be curved or that the appearance of objects moving about in an irregularly curved space might change had a natural appeal to early modern artists. The existence of curved space would necessarily invalidate the linear perspective system, whose dominance since the Renaissance was being challenged by the end of the nineteenth century. Likewise, traditional means of rendering objects could hardly be adequate if no absolute, unchanging form for an object could be posited. Important philosophical consequences also attended the birth of these new geometries. The proof of the fallibility of Euclid could only add to the growing recognition in the nineteenth century of the relative nature of the mathematical or scientific "truths" that man can discover.

The Geometry of *n* Dimensions

The development of *n*-dimensional geometry was far less unified, for no single discoverer of the geometries of four or more dimensions can be named. Instead, *n*-dimensional geometry emerged gradually during the second quarter of the nineteenth century, as a natural extension of analytic geometry, in which one or more variables are easily added to *x*, *y*, and *z*. Two important early works on this subject can be cited nevertheless: Arthur Cayley's 1843 article in the *Cambridge Mathematical Journal*,

involves its reevaluation in the light of projective geometry, a more general geometry that is freed of references to measurement. One of its proponents, Felix Klein, not only discovered an alternate model for Riemann's geometry, but also was able to establish both non-Euclidean geometry and Euclidean geometry as special cases within the overall unity of projective geometry (Boyer, *A History of Mathematics*, p. 592).

[10] This defect in Euclid's *Elements* was corrected only in 1899 in David Hilbert's critical study of the axioms of Euclid, *Grundlagen der Geometrie*, a work that focused further attention on alternatives to the Euclidean system.

"Chapters in the Analytical Geometry of *n* Dimensions," and Hermann Grassmann's *Die lineale Ausdehnungslehre* of 1844.[11] Riemann's 1854 speech with its broad view of a non-Euclidean geometry of *n* dimensions also contributed to the history of *n*-dimensional geometry and serves as a reminder that the two geometries can be joined, although they are two distinct branches of mathematics.

For much of the third quarter of the nineteenth century, when the move to *n* dimensions was made by various mathematicians, it was done simply as an extension of their own work on a particular type of problem. Thus, until the 1870s *n*-dimensional geometry remained largely an accessory to other geometrical research without a unified body of principles and theorems of its own. However, in 1870 Cayley set forth the general principles of *n*-dimensional geometry in his "Memoir on Abstract Geometry." And by 1881 Veronese could employ the methods of synthetic geometry for *n* dimensions, marking a shift toward more concrete representations of the subject matter of *n*-dimensional geometry.[12]

Although Möbius in 1827 had suggested that three-dimensional figures could be made to coincide if space had four dimensions,[13] interest in the physical properties of four-dimensional figures and space developed only gradually among mathematicians. Obviously, it was far more difficult to apply the physically oriented geometrical reasoning of synthetic geometry to higher dimensions than to treat the fourth dimension as an algebraic variable of analytic geometry. The initial challenge felt by nineteenth-century geometers would confront all successive advocates of the fourth dimension: how can we visualize a new dimension, perpendicular to each of the three dimensions of our familiar world?

N-dimensional geometry requires a redefinition of many of our common conceptions of geometrical principles. With the addition of a fourth dimension, new definitions of parallelism and perpendicularity must be made for "hyperspace," the space of four or more dimensions.[14] Often the four-dimensional property will be analogous to its three-dimensional counterpart with one dimension added. Thus, rotation in four dimensions occurs about a plane instead of about a line. The most obvious example of the operation of the rule of analogy is in the case of four-dimensional "hypersolids." A "hypercube" would be generated by the motion of a cube into a new fourth direction,

[11] Boyer, *A History of Mathematics*, p. 584. For a general history of *n*-dimensional geometry and an introduction to its principles, see Henry Parker Manning, *Geometry of Four Dimensions* (New York: Macmillan Co., 1914).

[12] Ibid., p. 8; Cayley, "Memoir," in *Philosophical Transactions of the Royal Society* (London), CLX (1870), 51-63; Veronese, "Behandlung der projectivischen Verhältnisse der Räume von verschiedenen Dimensionen . . ." in *Mathematische Annalen* (Leipzig), XIX (1881), 161-234.

[13] Möbius, *Der barycentrische Calcul* (Leipzig, 1827), p. 184; Manning, *Geometry of Four Dimensions*, p. 4.

[14] See Manning, *Geometry of Four Dimensions*, chs. 2, 4.

a process analogous to the generation of a cube by a square moving perpendicularly to itself. Similarly, a hypersolid is bounded by three-dimensional solids, just as the three-dimensional solids we know are bounded by two-dimensional planes. Such a complex figure must necessarily be viewed in sections either by passing it through our space so that new three-dimensional sections continually appear or by turning it on an axis and taking successive three-dimensional views of it.

A landmark in the study of hypersolids was the publication by W. I. Stringham of "Regular Figures in *n*-Dimensional Space" in the *American Journal of Mathematics* for 1880. The impact of this article was remarkable: there are numerous references to it in the writings of mathematicians and nonmathematicians in the early twentieth century, even after the combined use of synthetic and analytic methods had led to more sophisticated means of portraying four-dimensional hypersolids. Stringham worked purely synthetically, extending Euler's formula for polyhedra to establish the number of vertices, edges, and faces of the six regular polyhedroids of four-dimensional geometry.[15] He then produced one of the earliest known sets of illustrations of these hypersolids, depicting a summit of each of these figures spread out in three-dimensional space (Figs. 3, 4). Stringham's only attempts at any sort of complete view of any of the polyhedroids are his figures 2, 4, and 6, which are the complete projections on a plane of his figures 1, 3, and 5. His figure 4 is particularly noteworthy for its method of representing the four-dimensional hypercube, a depiction that was to be adopted by many popularizers of "the fourth dimension."

A mathematician who acknowledged the influence of Stringham's figures was Victor Schlegel, whose numerous articles on *n*-dimensional geometry during the 1880s and 1890s were published in German, French, and Italian mathematical periodicals. Schlegel's approach was more analytic than Stringham's, but in terms of the concrete representation of *n*-dimensional polyhedroids, Schlegel went even further than Stringham. He produced actual models of the polyhedroids' projections on three-dimensional space, exhibiting these for the first time at a congress of German physicians at Magdeburg in 1884.[16]

The absence of illusionism (and thus the possible suggestion of added dimensions) that plagues any portrayal of the fourth dimension by sculptural three-dimensional

[15] "A *regular polyhedroid* consists of equal regular polyhedrons together with their interiors, the polyhedrons being joined by their faces so as to enclose a portion of hyperspace, and the hyperplane angles formed at the faces by the half-hyperplanes of the adjacent polyhedrons being all equal to one another" (Manning, *Geometry of Four Dimensions*, p. 289). See Stringham, "Regular Figures in *n*-Dimensional Space,"*American Journal of Mathematics*, III (1880), 1-12.

[16] Maurice Boucher, *Essai sur l'hyperespace: Le Temps, la matière et l'énergie* (Paris: Félix Alcan, 1903), p. 140. Boucher also indicates that Schlegel's models were available commercially on a limited scale: "On peut les trouver à la librairie Martin Schilling à Halle-sur-Saale" (p. 140).

solids is likewise a problem in the diagram of the eight component cubes of the hypercube that had come into use by the end of the nineteenth century (Fig. 5). This version and its more popular, symmetrical counterpart (with the eighth cube omitted) are too-literal embodiments of the fact that four-dimensional hypersolids are bounded by three-dimensional solids. Here the danger for the unknowing observer is the false sense of completeness and adequacy, with no reminder that an entire dimension (the fourth) is missing. Slightly more correct is Figure 6, a three-dimensional perspective projection of the hypercube. However, in four-dimensional space each of the truncated pyramids surrounding the central cube would naturally retain its cubic shape, as the cubes here do not.

The basic difficulty is inherent in our conventional perception of the world. Visualizing a fourth perpendicular "inserted" into the intersection of the three dimensions that meet in the corner of a room seems impossible. Yet, as will be seen, advocates of a fourth dimension took great comfort in the analogy of a flat, two-dimensional world of beings unaware of the third dimension. The very challenge of visualizing and depicting higher dimensions, coupled with the encouraging message of the two-dimensional analogy, was certainly a major cause for the lasting fascination of the fourth dimension. But the difficulty of conceiving a fourth dimension also led to the occasional use of the more easily understood idea of time as the fourth dimension.

The first published suggestion that time be considered a fourth dimension was apparently made by d'Alembert in his 1754 article on "Dimension" in the *Encyclopédie* edited by Diderot and himself. There d'Alembert attributed the idea to "un homme d'esprit de ma connaissance." This "homme d'esprit" may well have been Lagrange, who is usually credited with this thought, although his first published reference to it occurred only in 1797 in his *Théorie des fonctions analytiques*.[17]

In the end, the definition of the fourth dimension as time was actually to displace popular interest in higher spaces. Following its use by H. G. Wells in his science fiction tale of 1895, *The Time Machine*, a temporal fourth dimension became part of the science "fact" of Minkowski's space-time continuum for Einstein's Theory of Relativity in 1908. However, in the late nineteenth- and early twentieth-century literature on the fourth dimension, time was always the less important of the two interpretations of the fourth dimension. If, in certain more philosophical and mystical expositions of a spatial fourth dimension, time played a role in the process of visualizing a higher dimension of space, time itself was not interpreted as the fourth dimension.

[17] On d'Alembert, see R. C. Archibald, "Time as a Fourth Dimension," *Bulletin of the American Mathematical Society*, xx (May 1914), 409-12; and d'Alembert, "Dimension," in *Encyclopédie*, ed. Diderot and d'Alembert (Paris: n.p., 1751-1765), IV, 1010. On Lagrange, see Manning, *Geometry of Four Dimensions*, p. 4.

It was the geometry of higher dimensions of space, along with non-Euclidean geometry, which fascinated the public in the early twentieth century.

The Rise of Popular Interest in the New Geometries

Ideas deriving from the new geometries, which had been the province of mathematicians alone in the first half of the nineteenth century, gradually began to appear in nonmathematical literature from the 1860s onward. This process is recorded in the listings of popular articles in Duncan Sommerville's *Bibliography of Non-Euclidean Geometry, Including . . . Space of n Dimensions.* Two specific areas of philosophical debate were the initial sources of public interest in non-Euclidean geometry and the geometries of higher dimensions: the nature of geometrical axioms and the nature of our space. Controversy about the nature of geometrical axioms naturally resulted from the challenge non-Euclidean geometry posed to Kant's view that the axioms of mathematics were a priori, and thus this debate concerned itself primarily with non-Euclidean geometry. The examination of the nature of our space, however, encompassed two questions: (1) the possible curvature of space, an issue relating to non-Euclidean geometry and the problem of geometrical axioms; and (2) the number of dimensions of space, suggested by geometries of higher dimensions. Certainly these two facets of the nature of space overlapped somewhat, and non-Euclidean geometry and *n*-dimensional geometry were linked at times in certain popular treatments. However, on the whole, the dispersion of the ideas of non-Euclidean geometry occurred in relation to geometrical axioms and the curvature of space, while that of *n*-dimensional geometry took place initially through writings on the number of dimensions of space.

Hermann von Helmholtz, through his popular writings on the axioms of geometry and the possible curvature of space, was a primary vehicle for disseminating these first ideas about non-Euclidean geometry. Helmholtz's articles, published not only in Germany but also in England, France, and the United States, attracted attention wherever they appeared during the 1860s and 1870s. However, no real upsurge of national interest can be discovered until the late 1880s and 1890s in Paris, when a heated debate was carried on in the *Revue Philosophique* and the *Revue de Métaphysique et de Morale*. This French controversy over the nature of geometrical axioms stands out in Sommerville's bibliography as the most concentrated nonmathematical manifestation of non-Euclidean geometry in the nineteenth century.

If the development of non-Euclidean geometry was a major impetus for a rethinking of questions about space, the idea of higher dimensions of space quickly assumed its own life, ultimately becoming far more popular than the notion of curved space. Sommerville's listings reveal that this fascination with higher dimensions first emerged in England during the 1870s, where "the fourth dimension" soon became a code name for *n*-dimensional geometry. A variety of connotations were gradually added to the geometrical meaning of "the fourth dimension," so that by 1900 the term had philosophical, mystical, and pseudoscientific implications along with its alternative interpretation as time. Although the early contributors to a lore of "the fourth dimension" were English, by the end of the nineteenth century the concept had gained publicity elsewhere through speculation on the nature of space in an increasing number of popular treatments. In the United States the fourth dimension was the subject of articles in magazines such as *The Popular Science Monthly* and *Science*. In France the question of the number of dimensions of space was frequently raised as part of contemporary investigations into space and our perception of it, and important statements on the subjects were made by Henri Poincaré, whose popular texts were read by a number of French artists.

The Popularization of Non-Euclidean Geometry: Helmholtz to Poincaré

When Kant in the *Critique of Pure Reason* of 1781 made his distinction between synthetic and analytic judgments and argued for the existence of judgments that are synthetic a priori, he offered the axioms of pure mathematics and geometry as his primary examples. According to Kant, the axiom "That the straight line between two points is the shortest" must be synthetic because its predicate is not contained in its subject (as in an analytic judgment), and a priori because "[it] carries with [it] necessity, which cannot be derived from experience."[18] The a priori nature of the axioms of geometry depended upon Kant's new definition of space in the *Critique* as a pure form of sensibility, which exists in the mind a priori as the frame in which all experience occurs. He writes in the *Critique*,

> Were this representation of space a concept acquired *a posteriori*, and derived from outer experience in general, the first principles of mathematical determination would be nothing but perceptions. They would therefore all share in the contingent character of perception; that there should be only one straight line between two points would not be necessary, but only what experience always teaches. What is derived from experience has only comparative universality, namely, that which is obtained through induction. We should therefore only be

[18] Immanuel Kant, *Critique of Pure Reason*, trans. Norman Kemp Smith (London: Macmillan & Co., 1929), p. 52.

> able to say that, so far as hitherto observed, no space has been found which has more than three dimensions.[19]

For Kant, "geometry" meant Euclidean geometry, the only geometry known for two thousand years. And in Kant's transcendental idealism, "space" was Euclidean space, possessing of necessity three dimensions.

The Kantian system was seriously challenged when Gauss, Lobachevsky, and Bolyai proved that certain of Euclid's axioms might be denied and a consistent geometry could still be worked out. The axioms of Euclid can hardly be said to originate within us, a priori, if we can conceive other systems of geometry. But how, then, did Euclid's system of axioms develop? John Stuart Mill, in his *System of Logic* of 1843, had already suggested that the elementary truths of geometry are simply the results of repeated observation.[20] With the rise of interest among mathematicians in the non-Euclidean geometries, Helmholtz seized upon the existence of these geometries as proof of his belief in the empirical origin of geometrical axioms and the impossibility of their being a priori.

Helmholtz produced many articles on the axioms of geometry, beginning in 1866 and culminating in his lecture "On the Origin and Significance of Geometrical Axioms." First given as a speech in Heidelberg in 1870 and published in *Academy* in London that year, Helmholtz's text was augmented for publication in *Mind* (London) in 1876. This final version was later incorporated in the second volume of Helmholtz's *Popular Lectures on Scientific Subjects* of 1881. To convey his message to a nonmathematical audience, Helmholtz employed the example of an imaginary world of two-dimensional beings living on the surface of a sphere, a model he most likely derived from Gauss. When it came to setting up a system of geometrical axioms, these reasoning beings would know nothing of parallel lines, for all their lines would intersect when extended sufficiently. Similarly, triangles would have angle sums greater than 180°, as they do in Riemann's geometry. "Nor are more examples necessary," writes Helmholtz, "to show that geometrical axioms must vary according to the kind of space inhabited by beings whose powers of reasoning are quite in conformity with ours."[21]

[19] Ibid., pp. 68-69.

[20] For Lobachevsky's conscious opposition to Kantian philosophy, see Vucinich, "Lobachevskii," pp. 473-76. On Mill's empirical view of mathematics, see Leszek Kolakowski, *The Alienation of Reason: A History of Positivist Thought*, trans. Norbert Guterman (Garden City, N.Y.: Doubleday, 1968), pp. 80-81.

[21] Helmholtz, "On the Origin and Significance of Geometrical Axioms" (1870), in *Popular Lectures on Scientific Subjects*, trans. E. Atkinson, 2nd ser. (London: Longmans, Green & Co., 1881), p. 38. Also included in this volume of *Popular Lectures* is Helmholtz's "On the Relation of Optics to Painting," which had been published in French translation in a book with E. Brucke's *Principes scientifiques des beaux-arts: Essais et fragments de théorie* in 1878. A list of the original versions and translations of Helmholtz's writings on geometrical axioms is included in the Bibliography, sec. I, B, 1.

The work of Helmholtz on the problem of geometrical axioms was one aspect of his fight against the nativist school of psychology rooted in the philosophy of Kant and Fichte. His overall concern was in combating the traditional Kantian idea of an innate a priori space with proof that our knowledge of space originates in experience. Helmholtz, thus, was one of the participants in the controversy between nativism and empiricism in psychology during the second half of the nineteenth century. Philosophically, his position was a positivist one, and Helmholtz's views would be regarded as such by later popular idealist philosophers, who believed that an empirical proof of the non-Euclidean or four-dimensional nature of our space was not necessary in order for it to be so.[22]

In the positivist, empiricist approach, geometrical axioms and space are as inextricably bound together in their empirical nature as they are in Kant's view of their necessarily mutual a priority. If the principles of geometry are the result of the space in which they are developed, as Helmholtz argues for his sphere dwellers, then these axioms should be verifiable by empirical observation, a process that will at the same time establish the structure of space. The existence of non-Euclidean geometry suggested, even to its first theorists, that a test should be made on our space, which for so long had been universally accepted as Euclidean.

Both Gauss and Lobachevsky tried to test their new geometry against physical space to determine if, perhaps, space did have an element of non-Euclidean curvature that had not been apparent enough to affect the formulation of Euclid's system. While Gauss attempted to measure the angle sum of an immense triangle formed by three mountain peaks, Lobachevsky sought to determine the curvature of space by measuring the parallax of distant fixed stars. In Euclidean space the "space constant" discovered

[22] For Helmholtz's role in the history of psychophysics, see Edwin C. Boring, *A History of Experimental Psychology* (New York: The Century Co., 1929), ch. 14. For a discussion of the empiricism-nativism controversy, see Boring, *Sensation and Perception in the History of Experimental Psychology* (New York: Appleton-Century-Crofts, 1942), pp. 233-38. Helmholtz was willing to admit that a certain basic Kantian spatiality might be intrinsic to the mind, but he insisted that this spatiality must include the possibility of both Euclidean and non-Euclidean spaces, the choice between them to be made by experience.

Maurice Mandelbaum, in *History, Man and Reason: A Study of Nineteenth-Century Thought* (Baltimore: Johns Hopkins Press, 1971), analyzes the evolution of positivism in the nineteenth century and the important changes it underwent, enabling the positivist view of science finally to merge with the idealist tradition. Helmholtz is among the "critical positivists" who rejected the attempts of "systematic positivists" like Comte and Spencer to develop a system including all disciplines and preferred simply a positivist philosophy of science (pp. 10-20). The two groups were united, nevertheless, in their scorn of metaphysics and their belief in experimental science as the ideal form of knowledge. For Mandelbaum's discussion of Helmholtz, see pp. 16, 292-98. Helmholtz actually understood his critical positivism as a Kantian position. He and certain other positivists reached this conclusion by taking Kant's emphasis on the role of experience out of the context of his transcendental philosophy and adapting it to their own ends. See Kolakowski, *The Alienation of Reason*, p. 102.

by these astronomical calculations is zero, while it would be positive and finite in the space of Lobachevsky's geometry and negative in Riemann's space. Neither Gauss nor Lobachevsky found any deviation from Euclidean values, but Lobachevsky's results were later cast into doubt when further astronomical study proved that certain of his figures were incorrect.[23] Thus, by the time Helmholtz was writing on the subject of a possible curvature of space, he could state that empirical observation seemed to confirm the Euclidean axiom of parallels, but that Euclid's geometry could not be taken as absolute in any sense. Future research, employing "other than our limited base-lines, the greatest of which is the major axis of the earth's orbit,"[24] might well prove that space was in fact non-Euclidean.

In the eyes of those who supported Kant's view of geometrical axioms, non-Euclidean geometry was not as "legitimate" as Euclidean geometry, because the space it engendered was not "intuitive."[25] Helmholtz, on the contrary, held that the human mind could intuit, or represent to itself, non-Euclidean space. He defended his view by demonstrating such a process of intuition, using Beltrami's model for three-dimensional pseudospherical space. Carefully defining "to represent" as "the power of imagining the whole series of sensible impressions that would be had in such a case," he took further precautions and distinguished his imaginable non-Euclidean space from a fourth dimension he believed was impossible to represent. Helmholtz had quickly dismissed the fourth dimension in his discussion of a plane world on a sphere and its development of a non-Euclidean geometry. Of the sphere dwellers he had written,

> But they could as little represent to themselves what further spatial construction would be generated by a surface moving out of itself, as we can represent what would be generated by a solid moving out of the space we know. . . . Now as no sensible impression is known relating to such an unheard-of event, as the movement to a fourth dimension would be to us, . . . such a "representation" is as impossible as the "representation" of colours would be to one born blind, if a description of them in general terms could be given to him.[26]

The "series of sensible impressions" necessary in Helmholtz's approach for an intuition of three-dimensional pseudospherical space was provided by Beltrami's suggestion that the interior of a Euclidean sphere corresponds to this type of non-Euclidean space. The surface of the sphere represents infinitely distant points of pseudospherical

[23] On how the measurement of the parallax of stars may determine the structure of the universe, see Max Jammer, *Concepts of Space: A History of Theories of Space in Physics* (Cambridge, Mass.: Harvard University Press, 1954), pp. 146-48.

[24] Helmholtz, "On the Origin and Significance," p. 55.

[25] See, e.g., Kant's major French defender, Charles Renouvier, "La Philosophie de la règle et du compas," *L'Année Philosophique*, II (1891), 1-66.

[26] Helmholtz, "On the Origin and Significance," pp. 34-35.

space, by virtue of the diminution in size of the units of space as they approach the surface. The observer in the center of the sphere will never be able to reach the surface of the sphere, and the lines along which he moves toward the surface will never intersect, behaving just as their counterparts in pseudospherical space.[27]

Helmholtz's example did not convince the neo-Kantians, however, who argued that Beltrami's model provided no real intuition of pseudospherical space.[28] This argument was to be repeated many times during the nineteenth century by the advocates of Kant in their defense of a priori space and Euclidean geometry. The question of the nature of geometrical axioms had by no means been settled by Helmholtz, and a major controversy over this question surfaced in France during the late 1880s and 1890s, where intermediary and new positions on the subject made the situation even more complex.

In addition to the numerous books on the subject, the *Revue Philosophique* and the *Revue de Métaphysique et de Morale* served as the major forums for the French debate on the nature of geometrical axioms, indicating its highly philosophical orientation.[29] Of the participants, no contemporary spokesman defended the purely empirical view of Helmholtz against the counterattack of Kant's defenders. Instead, those who criticized the Kantian position also rejected aspects of Helmholtz's view and then proposed various new theories, which quickly established antagonisms among themselves. The importance of the French debate was twofold. It gave non-Euclidean geometry a currency in Paris among intellectuals, and out of this debate emerged the definitive statements on this subject by Henri Poincaré, the mathematician-scientist and writer who, more than any other individual, was responsible for the popularizaton of non-Euclidean geometry in Paris during the first decade of the twentieth century.

Poincaré actually asserted his ideas quite early in the course of the debate, although he hardly convinced the strong partisans of other positions, who continued to argue for another fifteen years at least. In 1887 Poincaré first published his theory that the axioms of geometry are neither synthetic a priori nor empirical, but are conventions, a view now generally accepted as the solution to the controversy. In "Sur les hypothèses fondamentales de la géométrie" Poincaré had argued,

> One may now ask what these hypotheses [axioms] are. Are they experimental facts, analytic judgments or synthetic *a priori* judgments? We must respond neg-

[27] Ibid., pp. 48-49, 60-61.

[28] Antonio Aliotta, *The Idealistic Reaction Against Science*, trans. Agnes McCaskill (London: Macmillan & Co., 1914), pp. 280-81.

[29] On these French sources, see Bibliography, sec. I, B, 3. For a fuller listing of relevant articles, and a discussion of the debate, see Henderson, "The Artist," pp. 35-38, 517-18. Bertrand Russell provides an overview of the French debate in *An Essay on the Foundations of Geometry* (Cambridge: The University Press, 1897), pp. 110-16. Russell's own philosophy of geometry emphasized the a priori deduction of logic in mathematics. Poincaré opposed Russell on this issue, ever emphasizing the importance of intuition in geometry.

atively to these three questions. If these hypotheses were experimental facts, geometry would be subject to unceasing revision and would not be an exact science; if they were synthetic *a priori* judgments or, even more, analytic judgments, it would be impossible to remove one and establish a system on its negation. . . .

Thus, the fundamental hypotheses of geometry are not experimental facts; it is, however, the observation of certain physical phenomena which accounts for the choice of certain hypotheses among all possible ones.

On the other hand, the group chosen is only more convenient than the others and one cannot say that Euclidean geometry is true and the geometry of Lobachevsky is false any more than one could say that cartesian coordinates are true and polar coordinates false.[30]

Poincaré produced numerous articles during the 1890s and three major books between 1900 and 1910 in which he reiterated his views on geometrical axioms, adding arguments for the conventionalist view of the nature of space suggested at the end of the passage quoted above. Already in 1891 he had formulated his famous illustration of the impossibility of proving the truth or falsity of the hypothesis that our space is Euclidean.[31] If an astronomical triangle were measured and a deviation from 180° found, either Euclidean geometry could be renounced or the assumption could be made that light travels in curved instead of straight lines. The adoption of this last assumption, which could never be disproved according to Poincaré, would simply be more "commode" or convenient than the rejection of Euclidean geometry. Poincaré buttressed his position as the decade progressed, emphasizing that measurement can never be made of space itself, but only of bodies within that space, bodies whose behavior during the course of measurement is by no means certain. By his convincing arguments that the question "Which is the true Geometry?" is meaningless, Poincaré resolved the nineteenth-century debate on this subject. Only the advances of twentieth-century physics have produced a modification of his strictly conventionalist view.[32]

[30] Poincaré, "Sur les hypothèses fondamentales de la géométrie," *Bulletin de la Société Mathématique de France*, xv (1887), 215.

[31] Poincaré, "Les Géométries non euclidiennes," *Revue Générale des Sciences Pures et Appliquées*, II (15 Dec. 1891), 774. For Poincaré's writings, see Bibliography, sec. I, B, 3.

[32] Poincaré's conventionalist view of the axioms of pure geometry has been largely retained, but with the development of Einstein's General Theory of Relativity, Euclidean geometry no longer seems the "most convenient" for physicists studying space. Physicists in the tradition of Einstein assert that a valid "physical geometry" can be applied to the space of the universe, which according to their theories is non-Euclidean. Based on the assumptions they set forth about the behavior of physical bodies, the character of the geometry of space can be established empirically with the same degree of certainty as other scientific "truths." On this question, see Jammer, *Concepts of Space*, pp. 170-71, and Carl G. Hempel, "Geometry and Empirical Science" (1945), in *Readings in the Philosophy of*

The philosophical impact of non-Euclidean geometry in the nineteenth century was far greater than simply its initial challenge to Kant. It substantially shook the foundations of mathematics and science, branches of learning that for two thousand years had depended on the truth of Euclid's axioms.[33] As a result, optimistic belief in man's ability to acquire absolute truth gradually gave way during the later nineteenth century to a recognition of the relativity of knowledge. Coming full circle from its early days as a tool of the empiricist, positivist Helmholtz, non-Euclidean geometry contributed substantially to the demise of traditional positivism. For certain artists in the early twentieth century, non-Euclidean geometry was to be synonymous with the rejection of tradition and even with revolution.

The Popularization of n*-Dimensional Geometry and the Fourth Dimension in England and France*

From a survey of Sommerville's *Bibliography of Non-Euclidean Geometry, Including . . . Space of* n *Dimensions*, England in the 1870s emerges as the first center of active concern with the number of dimensions of space. This decade of development culminated in 1884 in the publication of E. A. Abbott's *Flatland*, the first example of popular fiction about the fourth dimension. Abbott's tale is based on the premise that the meaning of the third dimension for a two-dimensional being compares to the meaning of the fourth dimension for us. As noted above, this notion had been mentioned, if negatively, by Helmholtz in his discussions of geometrical axioms. Yet, beyond Helmholtz's passing reference, where might the theologian and educator Abbott have encountered "the fourth dimension" and the associated analogy of a world of two dimensions?

The question of why space should have three dimensions goes back at least to Aristotle, who discussed it in his *De caelo*.[34] Leibniz proposed geometric necessity as the explanation: no more than three mutually perpendicular lines can meet at a point. Kant likewise regarded the three dimensions of space as a synthetic a priori proposition of geometry.[35] Yet, in several earlier writings he had referred to other possible spaces.[36] In "On the First Grounds of the Distinction of Regions of Space" of 1769, Kant

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Science, ed. Philip P. Wiener (New York: Charles Scribner's Sons, 1953), pp. 40-51.

[33] On this, see, e.g., Morris Kline, *Mathematics: A Cultural Approach* (Reading, Mass.: Addison-Wesley Publishing Co., 1962), pp. 572-77.

[34] Jammer, *Concepts of Space*, p. 172; on the history of interest in the number of dimensions of space, including that of Aristotle and Leibniz, pp. 172-82. See also Manning, *Geometry of Four Dimensions*, pp. 1-3, for other examples of early speculation on higher dimensions.

[35] Kant, *Critique of Pure Reason*, p. 70.

[36] See Jammer, *Concepts of Space*, pp. 174-75.

speculated that the fundamental difference between a right hand and a left hand must be due to their different orientation with respect to absolute space.[37] Later writers on the fourth dimension, following the lead of Möbius, would frequently use the three-dimensional symmetry of a pair of hands as an argument for a fourth dimension of space, through which one hand could be "turned" to coincide with the other.

But the fourth dimension of these early philosophers was only an unspoken implication, a far cry from its central importance in speculation about the nature of space in England during the 1870s and 1880s. The first actual use of the term *fourth dimension* appears to have been in seventeenth-century England in the circle of the Cambridge Platonists around Henry More. However, the fourth dimension described in More's *Enchiridion Metaphysicum* of 1671 was not really a dimension of physical space, but rather the location of the Platonic ideal.[38] As mentioned previously, d'Alembert and Lagrange spoke of "une quatrième dimension" in the eighteenth century, understanding it simply as time.

Gustav Theodor Fechner, writing under the pseudonym of "Dr. Mises" in *Vier Paradoxa* of 1846, may have published the first discussion of two-dimensional beings unaware of the third dimension that surrounds them, suggesting the analogy of our three-dimensional existence in a four-dimensional world. The two-dimensional "shadow man" that Fechner presented in his satirical essay "Space Has Four Dimensions" was a projection on paper made by a camera obscura.[39] Fechner also noted the tendency of such figures, unaware of a third spatial dimension, to identify the effects of motion perpendicular to their plane as time. This theme, the interrelationship of a fourth dimension of space with time, was to become a major element in the hyperspace philosophy developed by the Englishman Charles Howard Hinton in the 1880s.

However, a more direct impetus to the rise of English speculation on the number of dimensions of space in the late 1860s was the biography of Gauss published by Sartorius von Waltershausen in 1856, the year after the mathematician's death.[40] J. J.

[37] Jammer, *Concepts of Space*, p. 131. Kant also noted in this text that only a mirror can transform a right hand into a left hand. Martin Gardner discusses Kant's ideas and their four-dimensional implications in *The Ambidextrous Universe: Mirror Asymmetry and Time-Reversed Worlds*, 2nd rev. ed. (New York: Charles Scribner's Sons, 1979), ch. 17, "The Fourth Dimension."

[38] Richardson, *Modern Art and Scientific Thought*, p. 106. See also R. Zimmermann, *Henry More und die vierte Dimension des Raumes* (Vienna: C. Gerolds Sohn, 1881).

[39] See Fechner [Dr. Mises], "Der Raum hat vier Dimensionen," in *Vier Paradoxa* (Leipzig: Leopold Voss, 1846), pp. 15-40. Fechner's text is discussed by Alexander L. Taylor in *The White Knight: A Study of C. L. Dodgson* (Edinburgh: Oliver & Boyd, 1952), pp. 89-90.

[40] Although J.P.N. Land notes Fechner's text in his discussion of the two-dimensional analogy in "Kant's Space and Modern Mathematics," *Mind* (London), II (1877), 43, and according to Taylor (n. 39 above), the presence of Fechner's colleague Max Müller at Oxford would have made C. L. Dodgson and others there aware of *Vier Paradoxa*, the Gauss-Helmholtz two-dimensional analogy is actually closer to the geometric spirit of *Flatland* and the articles by English mathematicians that preceded it.

Sylvester in a December 1869 article in *Nature* recorded the statement by Gauss's biographer that

> . . . this great man was used to say that he had laid aside several questions which he had treated analytically, and hoped to apply to them geometrical methods in a future state of existence, when his conceptions of space should have become amplified and extended; for as we can conceive beings (like infinitely attenuated book-worms in an infinitely thin sheet of paper) which possess only the notion of space of two dimensions, so we may imagine beings capable of realising space of four or a greater number of dimensions.[41]

Gauss's two-dimensional analogy had been developed in the course of his study of surfaces of curvature, and it was probably from Gauss that Helmholtz derived his model of the two-dimensional sphere dwellers. Although Helmholtz's concentration was also on curved surfaces, he did introduce the topic with the general case of any solid body, flat or curved.

The notion of a flat surface with two-dimensional beings living in it had already been described in Sylvester's article of 1869 when Helmholtz's "The Axioms of Geometry" was published in a February 1870 issue of *Academy*. Sylvester's article was the first of a series of semipopular articles by scholars of higher dimensions to appear in England and had been prompted by a remark of the biologist Thomas Huxley that mathematics offered no chance for exercise of the imagination or for empirical verification. This challenge may explain the radical stance of Sylvester, who boldly advocated the existence of four-dimensional space:

> Dr. Salmon, in his extensions of Chasles' theory of characteristics to surfaces, Mr. Clifford, in a question of probability . . . and myself in my theory of partitions, . . . have all felt and given evidence of the practical utility of handling space of four dimensions, as if it were conceivable space. . . . If Gauss, Cayley, Riemann, Schalfli, Salmon, Clifford, Krönecker, have an inner assurance of the reality of transcendental space, I strive to bring my faculties of mental vision into accordance with theirs.[42]

Following Sylvester, G. F. Rodwell sought to illustrate how our spatial perceptions could be altered in "On Space of Four Dimensions," published in *Nature* in May 1873. Rodwell first described the gradual transformation of a man into a two-dimensional being and analyzed the more limited ideas of space which would result. He then suggested that if this process were to be reversed, using motion as the dimension-

[41] Sylvester, "A Plea for the Mathematician," *Nature* (London), I (30 Dec. 1869), 238.

[42] Ibid., note.

adding factor, our ordinary space could be extended. Rodwell introduced his exposition by quoting Sylvester's statement about the group of prominent mathematicians with an "inner assurance of the reality of transcendental space."[43]

One impetus for the increased interest of mathematicians such as Sylvester in *n*-dimensional space had undoubtedly been the initial publication in 1867 of Riemann's 1854 speech. William K. Clifford's translation of Riemann's speech appeared in *Nature* in 1873, and Clifford himself produced articles on the subject of curved spaces.[44] However, the discussion by Riemann of the possibility of non-Euclidean spaces of *n* dimensions may also have caused the confusion that soon developed, identifying non-Euclidean geometry with geometries of higher dimensions. In the sense that Euclid dealt with only three dimensions of space, one might term *n*-dimensional geometry "non-Euclidean," but this usage is never employed in mathematics or in knowledgeable lay treatments of the subject. *N*-dimensional geometry and non-Euclidean geometry are two separate geometries, which can be combined, but are never necessarily so. Nonetheless, a more specific connection between the curvature of non-Euclidean geometry and higher dimensions was raised at this time by Clifford, the English disciple of Riemann. Clifford's interpretation was probably the major source for the mistaken intermingling of the two geometries, which would occur at times in later popular writings.

Sylvester's 1869 article already referred to the research that Clifford was pursuing:

> It is well known to those who have gone into these views, that the laws of motion accepted as a fact suffice to prove in a general way that the space we live in is a flat or level space (a "homaloid"), our existence therein being assimilable to the life of the bookworm in a flat page; but what if the page should be undergoing a process of gradual bending into a curved form? Mr. W. K. Clifford has indulged in some remarkable speculations as to the possibility of our being able to infer, from certain unexplained phenomena of light and magnetism, the fact of our level space of three dimensions being in the act of undergoing in space of four dimensions . . . a distortion analogous to the rumpling of the page.[45]

The idea that a higher dimension is necessary for curvature to occur "into" it may have seemed a logical inference from the case of the two-dimensional page. Yet, the internal geometry of that two-dimensional space would remain two-dimensional. Only when we wish to represent such a space from the outside, or "embed" it in Euclidean space, is a higher dimension required.

[43] Rodwell, "On Space of Four Dimensions," *Nature*, VIII (1 May 1873), 9.

[44] Clifford's major articles of the 1870s are in the Bibliography, sec. I, B, 2. Clifford also discussed curved space in "On the Bending of Space," *The Common Sense of the Exact Sciences* (London: Kegan Paul, Trench & Co., 1886), pp. 215-26.

[45] Sylvester, "A Plea," note, p. 238.

Helmholtz himself had warned that "curvature" must be understood differently for three-dimensional non-Euclidean spaces: "To prevent misunderstanding, I will once more observe that this so-called measure of space-curvature is a quantity obtained by purely analytical calculation, and that its introduction involves no suggestion of relations that would have a meaning for sense-perception. The name is merely taken, as a short expression for a complex relation, from the one case in which the quantity designated admits of sensible representation" (e.g. from two-dimensional surface curvature).[46] Although Helmholtz was specifically interested in precluding any talk of a fourth dimension in connection with his models of non-Euclidean space, subsequent, less partisan authors affirmed his view. Thus, Duncan Sommerville explained in his 1914 text *The Elements of Non-Euclidean Geometry*, "The use of the term 'space-curvature' has led to the idea that non-euclidean geometry of three dimensions necessarily implies space of four dimensions, for curvature of space has no meaning except in relation to a fourth dimension. But when we assert that space has only three dimensions, we thereby deny that space has four dimensions. . . . The origin of the fallacy lies in the failure to recognise that the geometry on a curved space is nothing but a *representation* of the non-euclidean geometry."[47] And in his 1897 *Essay on the Foundations of Geometry* Bertrand Russell had suggested the use of the phrase *space constant* instead of *measure of curvature*, as a reminder of the analytical nature of this idea.[48] However, despite the denials by Helmholtz and others of a necessary connection between non-Euclidean geometry and the fourth dimension, the ideas would continue to be linked at times in popular literature.

Besides the English mathematicians writing in the 1870s, another source for Abbott's thinking about two-dimensional beings could have been a pamphlet by the Reverend Charles L. Dodgson, better known as Lewis Carroll. A lecturer in mathematics at Oxford, Dodgson had introduced his 1865 text *Dynamics of a Parti-cle* by positing a romance between a pair of linear, one-eyed creatures gliding over a flat surface. According to Dodgson, the two young lovers know that, having been intersected by a line "making the two interior angles . . . less than two right angles," they "shall at length meet if continually produced."[49] Although Dodgson himself may have been stimulated by Fechner's *Vier Paradoxa* example, he was hardly advocating higher di-

[46] Helmholtz, "On the Origin and Significance," pp. 47-48.

[47] Sommerville, *The Elements of Non-Euclidean Geometry* (London: G. Bell & Sons, 1914), pp. 16-17.

[48] Russell, *Foundations of Geometry*, pp. 16-17. For other twentieth-century discussions of this problem, see Henry Parker Manning, ed., *The Fourth Dimension Simply Explained: A Collection of Essays Selected from Those Submitted in the "Scientific American" 's Prize Competition* (New York: Munn & Co., 1910), p. 12; d'Abro, *The Evolution of Scientific Thought*, ch. 5; and Stephen F. Barker, *Philosophy of Mathematics* (Englewood Cliffs, N.J.: Prentice-Hall, 1964), p. 37. It should be remembered that Einstein's space-time continuum is itself a four-dimensional non-Euclidean structure, with no talk by Einstein of its being "curved into" spaces of higher dimensions.

[49] Dodgson, *The Dynamics of a Parti-cle, with an Ex-*

mensions at this stage of his career. A conservative in mathematics, Dodgson in his *Dynamics of a Parti-cle* introduction was primarily concerned with Euclid's parallel postulate. Similarly, his subsequent exploration of mirror images and symmetry in *Through the Looking Glass* of 1872, with their four-dimensional implications, stands as a comment on contemporary English fascination with higher dimensions rather than a sign of his own belief in the idea.[50]

If Dodgson simply found humorous possibilities in a fourth dimension and Helmholtz denied in his 1876 *Mind* article that an individual could ever "represent" the fourth dimension to himself, the reality of higher dimensions was soon being proclaimed in London by J.C.F. Zöllner, a physicist and astronomer from Leipzig. Zöllner had been convinced of the existence of the fourth dimension by the feats of the American medium Henry Slade. A number of later authors have credited Zöllner with the single-handed popularization of the fourth dimension,[51] but this view is oversimplified. Nevertheless, the controversy generated by Zöllner's support of Slade in London brought the fourth dimension such notoriety that later defenders of higher dimensions of space often felt it necessary to disassociate themselves from Zöllner and the spiritualist connection.

Zöllner published two works in London: an article, "On Space of Four Dimensions," in *The Quarterly Journal of Science* (April 1878) and an 1880 translation of the third book of his *Wissenschaftliche Abhandlungen* (1878), *Transcendental Physics*. The article concludes with a specific statement of support for Slade against his accusers, who had recently had him convicted of "using 'subtle crafts and devices, by palmistry and

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cursus on the New Method of Evaluation, as Applied to π (Oxford: J. Vincent, 1865), p. iii. Dodgson's text, the main body of which has little relation to the theme of the introduction, is discussed in *The Magic of Lewis Carroll*, ed. John Fisher (New York: Bramhall House, 1973), pp. 10-11, as well as Taylor, *The White Knight*, pp. 68-69, 93. See again n. 40 above on Dodgson and Fechner.

50 Dodgson's conservatism in mathematics is apparent in such texts as *Euclid and His Modern Rivals* (1879) and *Curiosa Mathematica* (1888). On Dodgson, mirrors, and symmetry, see Martin Gardner, ed., *The Annotated Alice: "Alice's Adventures in Wonderland" and "Through the Looking Glass" by Lewis Carroll* (New York: Bramhall House, 1960), pp. 180-83, as well as Gardner, *Ambidextrous Universe*, pp. 5, 69, 114. Authors closer to Dodgson's era recognized more readily the four-dimensional overtones of his fiction. For example, Samuel M. Barton, writing in *The Popular Science Monthly* in October 1913, states, "Readers of the classic nonsense book by Lewis Carroll (Rev. C. L. Dodgson), 'Alice Behind the Looking Glass,' will be interested in the fact that Mr. Dodgson, himself a mathematician of no mean note, is poking fun at the fourth dimension students" (p. 388). Later in his life Dodgson mellowed in his attitude toward the occult (and, presumably, the fourth dimension) and in 1883 was included on the first list of members of the newly formed Society for Psychical Research (Taylor, *The White Knight*, p. 179).

51 See especially Hermann Schubert, "The Fourth Dimension" (1893), in *Mathematical Essays and Recreations*, trans. Thomas J. McCormack (Chicago: Open Court, 1898), pp. 64-111. Several of the essays submitted in the *Scientific American* essay contest of 1909 also discuss Zöllner in these terms. See, e.g., Louis W. Worrell, "Characteristics of the Fourth Dimension," in *The Fourth Dimension Simply Explained*, ed. Manning, p. 145.

otherwise,' to deceive Professor E. Ray Lankester, F.R.S., and certain others."[52] For English advocates of spiritualism, such as Sir William Crookes, the editor of *The Quarterly Journal of Science*, and C. C. Massey, who translated *Transcendental Physics*, Zöllner's "scientific" insights were most welcome.

Zöllner's union of spiritualism and science to prove empirically the existence of a fourth dimension was unusual in the history of the concept. Unlike later mystically oriented supporters of the fourth dimension who scorned positivist science for its limitation to immediately observable phenomena, Zöllner managed to encompass both points of view. While maintaining elsewhere that the fourth dimension might be the location of Kant's unknowable "thing-in-itself,"[53] Zöllner concluded his article's introductory argument by asserting, "It follows that the *real* existence of four-dimensional space can only be decided by *experience*, *i.e.*, by observation of *facts*."[54]

Zöllner's "facts" were provided by the experiments he conducted with the medium Slade in 1877. The scientific import of those experimental séances was enhanced by the presence of several of Zöllner's colleagues, including Fechner and Wilhelm Weber, all of whom he describes as "perfectly convinced of the reality of the observed facts, altogether excluding imposture or prestidigitation." The experiment which assured Zöllner that a higher dimension of space existed was Slade's untying a knotted cord whose ends were sealed together, without ever touching the cord. According to Zöllner, a fourth dimension of space must be posited to account for such a seemingly impossible occurrence: "The moment we observe in three-dimensional space contradictory facts,—*i.e.*, facts which would force us to ascribe to a body two attributes or qualities which hitherto we thought could not exist together,—the moment, I say, in which we should observe such contradictory facts in a three-dimensioned body, our reason would at once be forced to reconcile these contradictions."[55]

Additional experimental séances were conducted, with Slade joining solid wooden rings together, transporting objects out of closed three-dimensional containers, and, as his most masterful act, obtaining writing on paper placed within two slates securely sealed together.[56] Zöllner was thoroughly convinced by Slade's feats, and without a doubt his enthusiastic reporting of their every detail made both friends and enemies for the fourth dimension in England. Although his results must be termed pseudo-

[52] Charles Carleton Massey, Introduction to Zöllner, *Transcendental Physics: An Account of Experimental Investigations from the Scientific Treatises of Johann Carl Friedrich Zöllner* (1878), trans. Massey (London: W. H. Harrison, 1880), p. xxix.

[53] L.-Gustave du Pasquier discussed this aspect of Zöllner's thought in his inaugural address at the University of Neuchâtel, 23 April 1912: *La Quatrième Dimension et le développement de la notion d'espace* (Neuchâtel: n.p., 1912), p. 12.

[54] Zöllner, "On Space of Four Dimensions," *The Quarterly Journal of Science*, n.s., VIII (Apr. 1878), 232.

[55] Ibid., pp. 235, 229.

[56] See Zöllner, *Transcendental Physics*, for a thorough description of all the séances with Slade.

scientific at best, Zöllner's activities focused the attention of many people on the fourth dimension, further paving the way for Abbott's *Flatland* of 1884.

In the light of these developments, E. A. Abbott's two-dimensional "flatland" can be recognized as the progeny of Gauss's original notion, popularized by Helmholtz and the English mathematicians writing in *Nature* and other magazines. As the subtitle of *Flatland: A Romance of Many Dimensions by a Square* suggests, the hero of Abbott's charming story is a Square, a commoner in Flatland where all the citizens are figures from plane geometry. The women are straight lines, while the men vary in social rank according to the number of their sides, progressing from lowly squares, through polygons of increasing importance, to perfect circles who are priests. In Flatland it is strictly forbidden to think, much less to speak, of a third dimension.

One night, Abbott's hero, the Square, is visited by a Sphere from the Land of Three Dimensions, who materializes within his home, appearing first as a point and then as ever-increasing circles as it passes into the plane of Flatland. When the Square is lifted up by the Sphere, he, too, can see the wonders of the three-dimensional world and is so impressed that he soon inquires of the Sphere,

> But my Lord has shown me the intestines of all my countrymen in the Land of Two Dimensions by taking me with him into the Land of Three. What therefore more easy than now to take his servant on a second journey into the blessed region of the Fourth Dimension, where I shall look down with him once more upon this land of Three Dimensions, and see the inside of every three-dimensional house, the secrets of the solid earth, the treasures of the mines in Spaceland, and the intestines of every solid living creature, even of the noble and adorable Spheres.[57]

Abbott reveals his own familiarity with certain aspects of n-dimensional geometry in his description of the Square's further pleading with the Sphere, who hardly appreciates the Square's new-found open-mindedness:

> Or if it indeed be so, that this other Space is really Thoughtland, then take me to that blessed Region where I in Thought shall see the insides of all solid things. There, before my ravished eye a Cube, moving in some altogether new direction, but strictly according to Analogy, so as to make every particle of his interior pass through a new kind of Space with a wake of its own, shall create a still more perfect perfection than himself, with sixteen terminal Extra-solid angles, and Eight solid Cubes for his Perimeter. And once there, shall we stay our upward

[57] [Edwin Abbott Abbott], *Flatland: A Romance of Many Dimensions by a Square* (London: Seeley & Co., 1884). Edition used is Boston: Roberts Brothers, 1885, p. 135.

> course? In that blessed region of Four Dimensions, shall we linger on the threshold of the Fifth, and not enter therein? Ah, no! Let us rather resolve that our ambition shall soar with our corporal ascent. Then, yielding to our intellectual onset, the gates of the Sixth Dimension shall fly open, after that a Seventh, and then an Eighth—[58]

Unfortunately for the Square, talk of the fourth dimension is as strictly outlawed in the Land of Three Dimensions as the discussion of a third dimension had been in Flatland. The angry Sphere hurls the Square back down to Flatland where he is ultimately imprisoned for his tales of a third dimension.

Abbott's humorous message to three-dimensional Spheres and people who refused to admit even the possibility of a fourth dimension was quite clear. *Flatland* achieved instant success, with a second edition in 1884 and nine successive reprintings by 1915. Although the book was not translated into French, it was known in Paris, for E. Jouffret discusses the two-dimensional analogy and cites *Flatland* in his 1903 *Traité élémentaire de géométrie à quatre dimensions*, a book known to Duchamp and certain of his Cubist friends.[59]

In addition to popular literature in the tradition of Abbott, three other uses of the fourth dimension contributed to the spread of knowledge about it in nineteenth-century England and elsewhere: (1) a type of popular philosophy concerned with the fourth dimension, which I term *hyperspace philosophy*;[60] (2) Theosophy; and (3) science fiction tales and fantasies by H. G. Wells and others.

Writers of hyperspace philosophy believe firmly in the reality of a fourth dimension of space, yet tend to oppose any form of positivism that requires empirical proof of its existence. Their underlying theme is generally that the answer to the evils of positivism and materialism is for man to develop his powers of intuition, in order to "perceive" the fourth dimension of our world, the true reality. Hyperspace philosophy is an idealist position, and its proponents frequently refer to Plato's world of ideas or Kant's unknowable noumenon, the "thing-in-itself." Just as the more mystical bent of mind characteristic of hyperspace philosophy owes something to Zöllner's connection of the fourth dimension with spiritualism, hyperspace philosophy later takes on elements of the occult and at times unites with Theosophy.

The first true hyperspace philosopher was the Englishman Charles Howard Hinton,

[58] Ibid., pp. 138-39.

[59] E[sprit Pascal] Jouffret, *Traité élémentaire de géométrie à quatre dimensions* (Paris: Gauthier-Villars, 1903), p. 187. Jouffret also mentions Abbott in his *Mélanges de géométrie à quatre dimensions* (Paris: Gauthier-Villars, 1906), note, p. 216. Artistic interest in Jouffret's *Traité* is discussed below in chs. 2 and 3.

[60] I find this a useful means for characterizing writers from Hinton to Bragdon and Ouspensky, as opposed to authors of more straightforward mathematical expositions of the fourth dimension.

who began in 1880 publishing a series of articles and books on "the new era of thought," as he described it. Hyperspace philosophy later reached its most developed stage in the writings of P. D. Ouspensky in prerevolutionary Russia and, in fact, a direct link can be made from Hinton, through Ouspensky, to the Russian Futurist painters and poets. Although Hinton's books seem not to have been translated into French, as they were into Russian, his ideas were also known in Paris. Several authors, from Jouffret in his 1903 geometry text to L. Revel in an article in *Le Théosophe* of 1911, discuss Hinton's views on the fourth dimension.[61] Nor was the American avant-garde untouched by Hinton's ideas after he settled in the United States in the 1890s. Before his death in 1907, Hinton had gained a loyal follower in Gelett Burgess, who in turn shared Hinton's views with Claude Bragdon, later America's foremost theorist on the fourth dimension.

Most of the appreciation for Hinton, however, came after his death. During his lifetime Hinton, who had been trained in mathematics and physics at Oxford, never achieved prominence equal to that of his father, the noted English surgeon and liberal thinker James Hinton. Following a brief teaching career in England, Hinton in 1887 left his homeland for Japan, where he served for several years as the headmaster of the Victoria Public School in Yokohama and later worked for the Bureau of Mines. In 1892 he and his family settled in the United States, and he subsequently taught mathematics at Princeton University from 1893 to 1897 and at the University of Minnesota from 1897 to 1900. During 1901 and early 1902 Hinton was employed at the United States Naval Observatory, hired undoubtedly through the efforts of its recently retired director, Simon Newcomb, who was a prominent American advocate of *n*-dimensional geometry. By June 1902 Hinton had taken a job in the United States Patent Office in Washington D.C., where he was to work until his death in April 1907.[62]

[61] Revel's article, "L'Esprit et l'espace: La Quatrième Dimension," *Le Théosophe*, III (16 Mar. 1911), 2, and other popular reflections of Hinton's ideas in Paris are discussed in ch. 2.

[62] Born in 1853, Hinton earned a B.A. at Oxford in 1877 and an M.A. in 1886, at which time he was listed as the Assistant Master of Cheltenham College (*Alumni Oxoniensis: The Members of the University of Oxford, 1715-1886*, 4 vols., II [London: Joseph Foster, 1888], 666. The best source of biographical information on Hinton is the obituary written by his friend Gelett Burgess, "The Late Charles H. Hinton: Philosopher of the Fourth Dimension and Inventor of the Baseball Gun," *New York Sun*, 5 May 1907, p. 8. See also the brief obituary in the *New York Times*, 2 May 1907, p. 11. See a recent M.A. thesis, Marvin H. Ballard, "The Life and Thought of Charles Howard Hinton" (Virginia Polytechnic Institute and State University, 1980), for a great deal of new information on Hinton, gleaned from sources such as certain publications of the University of Minnesota. For example, the Minnesota yearbook, *The Gopher*, XII (1899), 37, states that Hinton studied physics briefly in Berlin after completing his B.A. in 1876, suggesting that he might have encountered the ideas of Fechner, Helmholtz, and Zöllner in Germany. For details of Hinton's career, see Ballard, "The Life and Thought," pp. 14-17, 46, 49, 51, 73, 76-77, 85, 97.

Hinton's abrupt departure from England sometime after October 1886 was the result of his trial and three-

Letters from Hinton to the psychologist and philosopher William James, written during the years 1892 to about 1906, provide an important record of Hinton's thinking once he left England. James was one of the few American intellectuals interested in Hinton's views on the fourth dimension, and a tone of discouragement frequently manifests itself in the letters addressed to James. "Nobody here will print anything which I have written," Hinton wrote on 8 October 1892.[63] Hinton's letters also make clear his commitment to creating a physical method for perceiving higher spatial dimensions, as opposed to the far easier mathematician's technique of manipulating symbols. Hinton's belief in the possibility of enlarging man's "space sense" is a constant theme in his hyperspace philosophy.

Hinton's first article, "What Is the Fourth Dimension?," had been published in the *Dublin University Magazine* in 1880 and included a discussion of a two-dimensional world, which may have been yet another forerunner of Abbott's *Flatland.* Other versions of this basic exposition of the fourth dimension were printed separately in London in 1884 and incorporated into Hinton's volume of *Scientific Romances*, published in 1884-1885 and again in 1886. A later volume of *Scientific Romances* (1895) resembled the 1886 book in its combination of hyperspace-philosophy essays and fictional tales of the fourth dimension. Hinton had left England before his first major text, *A New Era of Thought*, was published there in 1888.[64] Once in the United States, Hinton published several articles on the fourth dimension, both mathematical and popular, as well as *The Fourth Dimension* of 1904. His final work was *An Episode of Flatland* (1907), an elaboration upon Abbott's theme with greater emphasis on drama and even definite religious overtones.[65]

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day imprisonment for bigamy at that time. Hinton had undoubtedly been affected by his father's advocacy of freer sexual mores and even polygamy, views that made James Hinton the center of a circle of freethinking followers. The younger Hinton had, in fact, married the daughter of a member of this circle, Mary Everest Boole, widow of the mathematician George Boole. Both mother and daughter remained loyal to C. H. Hinton after the events of October 1886; Mary Hinton declined to press charges against her husband and accompanied him to Japan and America, and Mary E. Boole continued as a vocal supporter of Hinton's theories on four-dimensional space. Furthermore, his sister-in-law, Alicia Boole, edited the manuscript for *A New Era of Thought* that Hinton left behind in England. On the James Hinton circle (which later included Havelock Ellis after the elder Hinton's death) and on Hinton's bigamous marriage, see James Webb's introduction to Hinton, *Scientific Romances*, reprint ed. (New York: Arno Press, 1976), as well as Ballard, "The Life and Thought," pp. 42-44.

[63] Hinton letter to James, 8 Oct. 1892, William James Archive, Houghton Library, Harvard University. Robert C. Williams first noted the existence of this correspondence in *Artists in Revolution*, p. 217, n. 18. The reviews of the 1904 *The Fourth Dimension* demonstrate the mixed reaction of Hinton's contemporaries to his ideas. Although Hinton added a page of favorable comments from reviews of the book to its second edition (1906), far less complimentary views had been expressed by Bertrand Russell in *Mind*, n.s., XIII (Oct. 1904), 268.

[64] See Alicia Boole and H. John Falk, Introduction to Hinton, *A New Era of Thought* (London: Swan Sonnenschein & Co., 1888), p. v.

[65] For a listing of Hinton's publications, see Bibliography, sec. I, B, 2. Ballard, "The Life and Thought," discusses several additional articles by Hinton on non-

Hinton set forth his hyperspace philosophy most completely in the two volumes *A New Era of Thought* and *The Fourth Dimension*. Whereas Kant's interpretation of space as the a priori framework necessary for all perceptions had frequently been felt as a negative restraint by later commentators, Hinton suggests that our very dependence on space has a favorable aspect.[66] If it is through spatial intuition that we apprehend the world, we can work specifically on that space sense and develop it in order to intuit new kinds of space. Unlike the geometers of the empiricist school who saw Kant discredited by non-Euclidean geometry, Hinton credits Kant with identifying space as our means of cognizing the world. The three-dimensional Euclidean form Kant assumed for space was simply a temporary plateau in the course of man's development of spatial intuition, and Hinton's system would now provide the means for the next step upward.

A New Era of Thought introduces Hinton's belief that the root of our limited space sense is in "self elements" in each of us which reinforce our traditional ways of seeing things (e.g., as up or down or as left or right). The desired "casting out of self" is to be achieved through the careful, selfless study of an arrangement of objects. Hinton had settled on blocks of multicolored cubes as the best tools for concentrated analysis, a convenient choice since the same cubes were the basis for his system of learning to visualize the four-dimensional hypercube, or "tesseract," as he called it.

Both *A New Era of Thought* and *The Fourth Dimension* are basically descriptions of Hinton's work with the tesseract, which is noteworthy as the first nongeometric attempt to portray the fourth dimension. Although Hinton did write several scholarly articles on *n*-dimensional geometry, his real interest was in teaching the public the nonmathematical system he had devised. The frontispiece of *The Fourth Dimension* (Fig. 7) was in color, to help readers visualize the intricate combinations of the cubes Hinton would form into even more complex arrangements than the tesseract. The preface to *A New Era of Thought* advertises that models of the colored cubes used in the exercises can be bought from the publisher, Swan Sonnenschein & Co., a virtual necessity since no color plates were provided in that volume.[67]

Hinton conceived the tesseract by means of the sections that would be formed when it passed through three-dimensional space. Just as a sphere passing through a plane produces a series of increasing and then decreasing circles, which would be experienced by a plane dweller as movement in time, Hinton's method produces a time-oriented vision of a four-dimensional body. The colored cubes already introduced are to be the "sections" of the four-dimensional hypercube. We are to "see" the sections of the tesseract as they pass through our space, and the patterns of changing colors are the

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geometric subjects ranging from his invention of a baseball pitching "gun" to "Yoga Philosophy."

[66] Hinton's views on Kant are set forth in the first chapters of *A New Era of Thought*.

[67] See Hinton, *A New Era*, p. vi.

means of recognizing the position of the tesseract and its component cubes at any moment. Hinton continually emphasized the crucial role of time in his system: "All attempts to visualize a fourth dimension are futile. It must be connected with a time experience in three space [three-dimensional space]."[68]

Hinton was convinced his system had the power to revolutionize modes of vision in contemporary society:

> Here, for the first time, the fact of the power of conception of four-dimensional space is demonstrated, and the means of educating it are given.
>
> And I propose a complete system of work, of which the volume on four space is the first instalment.
>
> I shall bring forward a complete system of four-dimensional thought—mechanics, science, and art. The necessary condition is, that the mind acquire the power of using four-dimensional space as it now does three-dimensional.
>
> And there is a condition which is no less important. We can never see, for instance, four-dimensional pictures with our bodily eyes, but we can with our mental and inner eye. The condition is, that we should acquire the power of mentally carrying a great number of details.
>
> If, for instance, we could think of the human body right down to every minute part in its right position, and conceive its aspect, we should have a four-dimensional picture which is a solid structure. Now, to do this, we must form the habit of mental painting, that is, of putting definite colours in definite positions, not with our hands on paper, but with our minds in thought, so that we can recall, alter, and view complicated arrangements of colour existing in thought with the same ease with which we can paint on canvas. This is simply an affair of industry; and the mental power latent in us in this direction is simply marvellous.
>
> In any picture, a stroke of the brush put on without thought is valueless. The artist is not conscious of the thought process he goes through. For our purpose it is necessary that the manipulation of colour and form which the artist goes through unconsciously, should become a conscious power, and that, at whatever sacrifice of immediate beauty, the art of mental painting should exist beside our more unconscious art.[69]

[68] Hinton, *The Fourth Dimension* (London: Swan Sonnenschein & Co., 1904; New York: John Lane, 1904), p. 207. Fechner preceded Hinton in noting the interconnection between time and space, and Hinton may well have known *Vier Paradoxa*. Hinton evidently visualized his tesseract in the form of the popular cruciform "hypercube" discussed earlier (Fig. 5). To Hinton's credit is his addition of at least twenty-six colors to each of the cubes, thereby reminding readers of the inadequacy of the simple line drawing of the hypercube. In the end, however, Hinton's system was extremely complex, and it seems probable that few individuals apart from the author himself ever achieved much success with it.

[69] Hinton, *A New Era*, pp. 86-87.

The proposed "complete system of four-dimensional thought—mechanics, science, and art," was never finished by Hinton, and further discussion of art is conspicuously absent from his writings. Although his use of the painting metaphor in this passage is indeed fascinating and his emphasis on the artist's mind parallels later Cubist thought, Hinton's influence was not primarily visual, a fact due undoubtedly to the difficult nature of the tesseract model system. Instead, Hinton's important contribution was his philosophical interpretation of the fourth dimension, which was accompanied by his suggestions of its meaning for a science freed from the constraints of traditional positivism.

The philosophic and scientific ideas about the fourth dimension that Hinton presented initially in his articles and books recur frequently in later popular treatments of the fourth dimension. Two chapters of *The Fourth Dimension* present the "History of Four Space," tracing evidences of belief in the existence of higher dimensions back beyond Plato and Aristotle. Plato's allegory of the prisoners chained in a cave, who saw only shadows on a wall and never knew of the beings creating those shadows, offered Hinton and others an even closer analogy to the question of higher dimensions than it had to Plato's world of ideas. The association of the fourth dimension with the Platonic ideal and, subsequently, with Kant's "thing-in-itself" was to become a standard feature of hyperspace philosophy. Unlike the vague fourth dimension of Platonist Henry More in the seventeenth century, the fourth dimension of hyperspace philosophy was specifically a dimension of space where the "thing-in-itself" would be revealed.

In the idealism of hyperspace philosophy it is the absence of the fourth dimension in perception that flaws our image of true reality and limits us to a three-dimensional world of appearances. For Hinton, "we must really be four-dimensional creatures or we could not think about four dimensions."[70] Perhaps, he suggests, our successive states are the passing of our four-dimensional being through the three-dimensional space in which our consciousness seems confined, or possibly our extension in the fourth dimension is so "thin" that it escapes our senses. According to Hinton and later writers, if a fourth dimension of space does exist, the three-dimensional world must possess a slight extension in this fourth dimension; otherwise, our world would be merely a geometrical abstraction analogous to a two-dimensional plane without even the thickness of a sheet of paper.[71]

Hinton believed that the idea of a small extension in four-dimensional space could be useful for scientists studying minute particles of matter, for in the infinitesimal the

[70] Ibid., p. 99.

[71] Hinton, "What Is the Fourth Dimension?" *Dublin University Magazine* (Dublin and London), XCVI (1880), 28, 33-34. See also "The Fourth Dimension," *Harper's Monthly Magazine*, CIX (July 1904), 230-31.

dimensions in all four directions would be more comparable. The one scientist Hinton actually discusses is Lord Kelvin, whose theory of atoms as vortex rings of the ether and other unusual notions fascinated many observers of science in the late nineteenth century. Although the fourth dimension had played no part in Kelvin's work with vortices, Hinton suggests that particles of ether in four dimensional movement would produce a vortex with characteristics similar to electric current. Thus, in Hinton's view electricity may well be a four-dimensional phenomenon, and he reinforces his theory by pointing out that positive and negative electric currents could only be made to coincide by moving them through a fourth dimension.[72] Hinton in the tradition of Kant also discusses organic, right- and left-handed symmetry as evidence that a higher dimension of space must exist.

Finally, in a broader view, Hinton proposes that the ether itself may be a surface of contact shared by two four-dimensional existences.[73] Just as two-dimensional beings could exist in the plane formed as the common boundary of two adjacent cubes, without ever perceiving their three-dimensional extensions, we may live in a three-dimensional ether generated through the contact of two four-dimensional bodies. Even if the majority of Hinton's hypotheses have survived no better than has the ether,[74] his theories had a widespread impact on early twentieth-century advocates of "the fourth dimension" and contributed substantially to the body of beliefs on the subject then being codified.

A second major impetus for a widening interest in higher dimensions of space was Theosophy. In 1888, the year in which Hinton's *A New Era of Thought* appeared, Helena Petrovna Blavatsky published in London *The Secret Doctrine*, her second major Theosophical treatise. Madame Blavatsky, the moving force behind Theosophy, had produced her first two-volume opus, *Isis Unveiled*, in 1877, but not until *The Secret Doctrine* does she refer even in passing to the fourth dimension. However, with her own Theosophical doctrines fully developed by this time, Madame Blavatsky found no place for the fourth dimension in Theosophy. Her position is in marked contrast to that of later Theosophists, writing after the fourth dimension had gained a degree of popularity. As early as the 1890s, Theosophy's antipositivist stance made it strikingly

[72] Hinton, *The Fourth Dimension*, pp. 17-18, 84. Gardner, *Ambidextrous Universe*, pp. 223-28, notes the remarkably prophetic nature of Hinton's ideas on positive and negative charges as mirror images. Even though Hinton's theory of electricity as a vortex in the ether was hardly accurate, certain of his conclusions about symmetry (e.g., "twists" and "image twists") sound much like mid-twentieth-century speculation in particle physics on matter and antimatter.

[73] Hinton, *A New Era*, p. 53.

[74] For the history of the "luminiferous ether," which passed out of the mainstream of scientific thought with the acceptance of Einstein's Special Theory of Relativity after World War I, see Sir Edmund Whitaker, *A History of the Theories of Ether and Electricity*, 2 vols. (London: Thomas Nelson & Sons, 1951, 1953). On the subsequent fate of the ether, see Appendix A, n. 28.

similar to hyperspace philosophy, and in 1895 the Theosophist C. W. Leadbeater would compare the Theosophical idea of "astral vision" to four-dimensional sight.[75]

Madame Blavatsky herself believed that the idea of a fourth dimension of space was misconceived:

> The processes of natural development which we are now considering will at once elucidate and discredit the fashion of speculating on the attributes of the *two, three,* and *four* or more "dimensional Space"; but in passing, it is worth while to point out the real significance of the sound but incomplete intuition that has prompted—among Spiritualists and Theosophists, and several great men of Science, for the matter of that—the use of the modern expression, "the fourth dimension of Space." To begin with, of course, the superficial absurdity of assuming that Space itself is measurable in any direction is of little consequence. The familiar phrase can only be an abbreviation of the fuller form—the "*Fourth dimension of* MATTER *in Space.*" But it is an unhappy phrase even thus expanded, because while it is perfectly true that the progress of evolution may be destined to introduce us to new characteristics of matter, those with which we are already familiar are really more numerous than the three dimensions. . . . [T]hus, when some bold thinkers have been thirsting for a fourth dimension to explain the passage of matter through matter, and the production of knots upon an endless cord, what they were really in want of, was a *sixth characteristic of matter.*[76]

Blavatsky cites Professor Zöllner in a footnote to this passage and his association with Theosophy was to continue. Wassily Kandinsky, for instance, owned a copy of Zöllner's *Die transcendentale Physik und die sogenannte Philosophie* and cited this book in his strongly Theosophically oriented text of 1912, *Concerning the Spiritual in Art.*[77] Madame Blavatsky notwithstanding, many Theosophists became actively interested in the fourth dimension. Like Kandinsky, František Kupka and Piet Mondrian were naturally sympathetic toward the fourth dimension because of their Theosophical beliefs. For Kupka, in fact, a number of contemporary sources in Paris emphasized the parallels between higher dimensionality and Theosophical doctrine. Even more importantly, two of the major hyperspace philosophers of the twentieth century, the American Claude Bragdon

[75] On Theosophy, see Bruce F. Campbell, *Ancient Wisdom Revived: A History of the Theosophical Movement* (Berkeley: University of California Press, 1980); for Leadbeater's discussion, see C[harles] W[ebster] Leadbeater, *The Astral Plane: Its Scenery, Inhabitants, and Phenomena* (1895) (London: Theosophical Publishing House, [1910]), p. 19.

[76] Blavatsky, *The Secret Doctrine* (London: The Theosophical Publishing Co., 1888; reprint ed. Los Angeles: The Theosophy Co., 1925), p. 251.

[77] Kandinsky, *Concerning the Spiritual in Art* (1912), retrans. Francis Golffing, Michael Harrison, and Ferdinand Ostertag (New York: Wittenborn, Schultz, 1947), p. 32.

and the Russian P. D. Ouspensky, came to the fourth dimension with backgrounds in Theosophy.

The final major vehicle for popularizing the fourth dimension in the late nineteenth century was the science fiction of H. G. Wells, who, with his French disciple Gaston de Pawlowski and certain other authors, found in the notion a particular appeal. Not only was it a popular fascination, but the idea of the fourth dimension as a place or as a temporal means of reaching another era provided a position from which to comment on contemporary society. This was clearly Wells's purpose in his 1895 tale *The Time Machine*, although other of his stories rely simply on the mysterious properties of four-dimensional space.

Wells is thought to have first encountered the idea of a fourth dimension while attending the Royal College of Science from 1884 to 1887.[78] Interest in higher dimensions among students at the Royal College is documented by a paper entitled "Fourth Dimension" by Wells's fellow student E. A. Hamilton Gordon, reprinted in the *Science Schools Journal* in April 1887. The next year Wells's prototype for *The Time Machine*, "The Chronic Argonauts," was published in the same journal. Early drafts of the book appeared serially in the *National Observer* in 1894 and in the *New Review* in early 1895.[79] In addition to his probable English sources, such as Hinton and Abbott, Wells by 1894 was also aware of the work of the American mathematician Simon Newcomb, who is cited in *The Time Machine*.[80]

In the tradition of d'Alembert and undoubtedly extrapolating from hyperspace philosophy, Wells treats time itself as the fourth dimension. However, his "Time Traveller" does discuss geometries of higher dimensions in explaining the fourth dimension to his friends at the beginning of the story. In support of his theory that it is possible to move about in time as one does in space, the Time Traveller asserts,

> Clearly, . . . any real body must have extension in *four* directions: it must have Length, Breadth, Thickness, and—Duration. But through a natural infirmity of the flesh, which I will explain to you in a moment, we incline to overlook this

[78] Bernard Bergonzi, *The Early H. G. Wells: A Study of the Scientific Romances* (Toronto: University of Toronto Press, 1961), pp. 25, 31. Bergonzi provides an account of Wells's introduction to the fourth dimension at South Kensington, although he seems unaware of the complex series of events behind the rise of "the fourth dimension" in England.

[79] Bergonzi reprints "The Chronic Argonauts" as an appendix to *The Early H. G. Wells*. For the *National Observer* and *New Review* versions, see *H. G. Wells: Early Writings in Science and Science Fiction*, ed. Robert M. Philmus and David Y. Hughes (Berkeley: University of California Press, 1975), pp. 57-61, 91-95. Philmus and Hughes provide further background for Wells's ideas and do cite Abbott and the 1884 printing of Hinton's "What Is the Fourth Dimension?" (pp. 47-49).

[80] Wells, *The Time Machine: An Invention* (London: W. Heinemann, 1895), p. 3. Philmus and Hughes cite Newcomb's article, "Modern Mathematical Thought," *Nature*, XLIX (1 Feb. 1894), 325-29, as Wells's immediate source (*H. G. Wells*, p. 49, n. 5).

fact. There are really four dimensions, three which we call the three planes of Space, and a fourth, Time. There is however, a tendency to draw an unreal distinction between the former three dimensions and the latter, because it happens that our consciousness moves intermittently in one direction along the latter from the beginning to the end of our lives.[81]

With a mysterious machine he has constructed, a device only vaguely described by Wells, the Time Traveller propels himself into the future, where he lands in the year 802701. The main narrative involves his gradual discovery of the nature of the society he finds there and, ultimately, his dramatic escape from it. Wells's story, influenced by aspects of Darwin and Marx, is essentially a comment on class struggle in the late nineteenth century.[82] In *The Time Machine* the fourth dimension proved its versatility, serving as a temporal vehicle that allowed Wells to express his social theory.

Wells employed a purely spatial fourth dimension in two other tales of 1895, "The Remarkable Case of Davidson's Eyes" and *The Wonderful Visit*. By means of a "kink in space" Davidson is able to observe events on an island in the South Seas while still in London. For this to occur, two three-dimensional worlds must be folded together in four-dimensional space just as corners of a two-dimensional napkin may be joined in three-dimensional space.[83] *The Wonderful Visit* is likewise based on the idea of separate adjacent three-dimensional worlds, but in this case an angel falls out of his heavenly world into an English country village.[84] By implication, the angel is also aware of the true four-dimensional nature of things and, much like the omniscient Sphere in *Flatland*, emphasizes by contrast the pettiness and limited outlook of the villagers.

In his 1896 "The Plattner Story" Wells relied on the frequently discussed notion

[81] From the hyperspace philosophy of Hinton and other contemporary sources Wells would have learned of the interrelationship of time and space in the fourth dimension. In fact, a brief note by "S.," published in *Nature* in 1885, had described explicitly the idea of time as a fourth dimension in which the motion of a cube would create a "sur-solid" in "time-space." See "S.," "Four-Dimensional Space," *Nature*, xxxi (26 Mar. 1885), 481. Interested in the fictional possibilities of time travel, Wells in this one instance resurrected the d'Alembertian definition of the fourth dimension as time itself. Wells's reference to "duration" suggests that time held a further appeal for him as an aspect of the contemporary theories of Henri Bergson. On Wells and Bergson, see Philmus and Hughes, *H. G. Wells*, pp. 48-49, n. 4. For Bergson's negative view of a spatial fourth dimension, see the section of Chapter 2 following n. 200.

In the version of *The Time Machine* published in *The New Review* Wells had included a discussion of a "Rigid Universe" with three dimensions of space and one of time. In this section, omitted from the final Heinemann text of *The Time Machine*, Wells was uncannily close to Minkowski's vision of an overall continuum of space-time that encompasses the "world-lines" of every individual. For this text, "The Inventor," see Philmus and Hughes, *H. G. Wells*, pp. 91-95.

[82] Bergonzi, *The Early H. G. Wells*, p. 46.

[83] Wells, "The Remarkable Case of Davidson's Eyes" (1895), in *The Country of the Blind and Other Stories* (London and New York: T. Nelson & Sons, [1913]), pp. 99-100. Hinton had discussed this property of higher space as early as 1880 in his article "What Is the Fourth Dimension?"

[84] Wells, *The Wonderful Visit* (London and New York: Macmillan & Co., 1895), p. 26.

of three-dimensional symmetry and an object's ability to turn through itself in the fourth dimension. Thus, Gottfried Plattner, an unfortunate science teacher blown away by an explosion of his chemicals, returns from a visit to a misty other world with his heart on his right side. Finally, the mad scientist who has become *The Invisible Man* (1897) owes his invisible status to "a formula, a geometrical expression involving four dimensions."[85] In this case Wells followed the lead of Hinton, who in 1895 had published a scientific romance about an invisible girl, *Stella*. Her invisibility had been produced by the reduction of her "coefficient of refraction" to zero, a process with strong overtones of the fourth dimension.

During the 1890s several of Wells's literary contemporaries working in Britain invoked the fourth dimension, further attesting to its popularity by this time. In Oscar Wilde's irreverent spoof of ghost stories, "The Canterville Ghost" of 1891, "the fourth dimension" is the target of Wilde's humor, along with members of the "Psychical Society" and all others who have ever taken supernatural phenomena seriously. Of his long-suffering ghost, whom the new American tenants of Canterville chose simply to take in stride, Wilde writes at one point, "There was evidently no time to be lost, so hastily adopting the Fourth Dimension of Space as a means of escape, he vanished through the wainscoting and the house became quiet." Charles Dodgson's friend George Macdonald used a spatial fourth dimension with heavenly associations as a central theme in his *Lilith* of 1895. Macdonald's Adam figure, Mr. Raven, travels back and forth from another world through a secret garret in which he arranges mirror reflections to produce an opening in space. Mr. Raven himself had learned of higher dimensions from "old Sir Up'ard," whom Macdonald may well have named as a result of the litany of the Square in *Flatland*, "Upward, yet not Northward," as he struggled to comprehend the third dimension. Joseph Conrad and Ford Madox Hueffer (later Ford Madox Ford) based their 1901 novel *The Inheritors* on a fourth dimension. Their "inheritors" were a superhuman, but cruel and unfeeling, race from the Fourth Dimension, who were gradually taking over the world.[86]

Among these authors, however, Wells was the writer most committed to the fourth dimension. Through the 1920s he continued to seek a literary "fourth dimension,"

[85] Wells, "The Plattner Story" (1896), in *The Country of the Blind*, pp. 307-8; Wells, *The Invisible Man: A Grotesque Romance* (London: C. A. Pearson, 1897), p. 144.

[86] Wilde, "The Canterville Ghost," in *Lord Arthur Savile's Crime and Other Stories* (London: James R. Osgood, McIlvaine & Co., 1897), reprinted in *The Works of Oscar Wilde*, 15 vols. (New York: Lamb Publishing Co., 1909), IV, 103-4; Macdonald, *Lilith: A Romance* (London: Chatto & Windus, 1895; New York: Dodd, Mead & Co., 1895), ch. 8; Abbott, *Flatland*, p. 141; Conrad and Hueffer, *The Inheritors: An Extravagant Story* (London: W. Heinemann, 1901). In contrast to the group of writers discussed here, Rudyard Kipling in his short story "An Error in the Fourth Dimension" uses the term offhandedly to emphasize the gravity of his hero's social error of stopping a train en route. See Kipling, "An Error in the Fourth Dimension," *The Cosmopolitan* (New York), XVIII (Dec. 1894), 212-21.

now reinterpreted by him as a transcendence of the conventions of fiction, in tune with the space-time world of Einstein's General Relativity.[87] At the turn of the century Wells was undoubtedly the most widely known figure to be associated with the fourth dimension. In the United States his books were often published simultaneously with the English first edition, and in France each of Wells's tales involving the fourth dimension, except "The Plattner Story," had been translated and published by the early years of the twentieth century.

"The fourth dimension" became popular considerably later in France than in England. By 1900, however, the concept had begun to emerge from more learned speculations about the nature of space, which were given a special impetus in France by the controversy over geometrical axioms. Just as Poincaré's writings on the nature of geometrical axioms were central to the popularization of non-Euclidean geometry, so his statements on the number of dimensions of space lent prestigious support to the cause of "the fourth dimension" in France.

Although in *De l'intelligence* of 1870 Hippolyte Taine had argued in a positivist vein against the existence of a fourth dimension of space,[88] and Helmholtz's denials of the possibility of representing the fourth dimension were known in France through translations of his works, complete denials of the possibility of higher dimensions were far more rare. An important factor in this more moderate attitude was undoubtedly the recognition that there is more than one kind of space: geometric space and perceptual space, or physiological space as Ernst Mach termed it, are separate entities. Mach had mentioned this distinction in his *Analysis of the Sensations* of 1886[89] and was to devote his 1906 book, *Space and Geometry*, to the study of physiological space. More importantly, in France this differentiation between the spaces of geometry and of perception was emphasized in the writings of Poincaré as early as 1895. Poincaré characterized geometric space as follows: "1. It is continuous; 2. It is infinite; 3. It has three dimensions; 4. It is homogeneous, that is to say, all points are identical one with another; 5. It is isotropic, that is to say, all the straights which pass through the same point are identical one with another." In contrast, perceptual space is made up of three component spaces, visual, tactile, and motor, which are neither continuous, infinite, homogeneous, nor isotropic. Of perceptual space, "one cannot even say that it has three dimensions."[90] For both Poincaré and Mach, a third type of space was also implied, the space of our universe beyond our immediate perception.

[87] On this subject, see William J. Scheick, "The Fourth Dimension in Wells's Novels of the 1920's," *Criticism: A Quarterly for Literature and the Arts*, xx (Spring 1978), 167-90.

[88] Jouffret, *Mélanges de géométrie à quatre dimensions*, p. 197.

[89] Mach, *Contribution to the Analysis of the Sensations* (1886), trans. C. M. Williams (Chicago: Open Court, 1897), p. 177.

[90] Poincaré, *La Science et l'hypothèse* (Paris: Ernest

In the light of this more sophisticated analysis of space, Helmholtz's single space, which gives rise through perception to a certain geometry that, in turn, reflects the geometry of the universe, is too simplistic. Poincaré's arguments against the possibility of an empirical proof of the Euclidean or non-Euclidean nature of the universe can now be seen to rest as well on this distinction of various types of spaces. Instead of geometric space being a direct result of our perceptions, as Helmholtz maintained, for Poincaré it is an idealized space based on axioms that are merely conventions, just as the geometrical model we adopt for the universe is a convention.[91] Poincaré's denial of an empirical proof for the Euclideanism or non-Euclideanism of the universe also applies to the question of how many dimensions it has, and thus leaves open the possibility of a four-dimensional world.

Poincaré's discussions of the fourth dimension dealt primarily with perceptual space, however. Here his conventionalist philosophy gave him a freedom disallowed by the strict positivism of Helmholtz, for whom the three dimensions of space were assured by the certainty of experience and a fourth dimension was unimaginable. Yet, in the words of Poincaré, "experience does not prove to us that space has three dimensions; it only proves to us that it is convenient to attribute three to it."[92] And furthermore, "a person who should devote his existence to it might perhaps attain to a realization of the fourth dimension."[93]

According to Poincaré, our notions of visual, tactile, and motor space are generated through associations among sensations, which are developed through personal experience and heredity. Because these associations have become customary, it is difficult, though possible, to break them apart. If, for instance, the two muscular sensations of accommodation and convergence of the eye, which normally function together in one series, were to vary independently of one another, the "complete visual space" to which they give rise would have four instead of three dimensions. Pursuing this line of thought, Poincaré makes a statement that must have intrigued the Cubists and their generation: "From this point of view, *motor space would have as many dimensions as we have muscles.*"[94]

Flammarion, 1902), pp. 69, 74; trans. George Bruce Halsted in Poincaré, *The Foundations of Science: Science and Hypothesis, The Value of Science, Science and Method* (New York: The Science Press, 1913), pp. 66-67, 70.

[91] For a summary of Poincaré's views on space and geometry, see especially Part 2, "L'Espace," of *La Science et l'hypothèse*. The ideas of this section had generally been published in earlier articles. For instance, ch. 4, "L'Espace et la géométrie," is comparable to the article of the same name in *Revue de Métaphysique et de Morale*, III (1895), 631-46.

[92] Poincaré, *La Valeur de la science* (Paris: Ernest Flammarion, 1904), p. 125; *The Foundations of Science*, p. 272.

[93] Poincaré, "Les Géométries non euclidiennes," *Revue Générale des Sciences Pures et Appliquées* (1891), p. 774. See also, *La Science et l'hypothèse*, p. 68; *The Foundations of Science*, p. 66.

[94] Poincaré, *La Science et l'hypothèse*, p. 73; *The Foundations of Science*, p. 69. See *La Science et l'hypothèse*, ch. 4, for Poincaré's argument as presented here.

Poincaré developed his theories on the process of spatial perception further in *La Valeur de la science* of 1904 and presented his most complete discussion in the 1908 *Science et méthode*. His concluding remarks in this text lent further support to the case for higher dimensional space: "So the characteristic property of space, that of having three dimensions, is only a property of our table of distribution, an internal property of human intelligence, so to speak. It would suffice to destroy certain of these connections, that is to say of the association of ideas to give a different table of distribution, and that might be enough for space to acquire a fourth dimension."[95]

After the publication of *Science et méthode*, however, Poincaré's views on the fourth dimension became more conservative. Through his work in *analysis situs*, the forerunner of modern topology, Poincaré believed he had found an explanation for the seeming necessity of three dimensions for the mathematical continuum (geometric space), as well as the physical continuum (perceptual space). Using the topological notion of "cuts," Poincaré pointed out that every continuum of *n* dimensions is cut or divided completely by a continuum of *n*-1 dimensions: a line by a point, a surface by a line, and a space by a surface. For space, the surface that cuts it into two absolutely separate parts has two dimensions, so space in this respect must have three dimensions.[96] Poincaré's conclusions to this effect were published in the *Revue de Métaphysique et de Morale* in 1912, the year of his death, and, posthumously, in *Dernières Pensées* of 1913. Despite these later conclusions, the more liberal views Poincaré had expressed in his writings of the first decade of the century provided vital support for Frenchmen interested in higher spatial dimensions.

Poincaré's conventionalist attitude toward spatial dimensions was part of an increasing recognition toward the end of the nineteenth century that sense perceptions were relative.[97] Ironically, for Poincaré and others the non-Euclidean geometry upon which Helmholtz had earlier based his empirical philosophy was an important support for the new relativist, antipositivist attitude toward spatial perceptions. The very model of two-dimensional beings on a sphere that Helmholtz had posited, now generalized to a two-dimensional *Flatland* in explanations of the fourth dimension, was employed to demonstrate the limits of sense perception. In the *Revue Scientifique*, for example, a May 1897 article by Sir William Crookes, "De la relativité des connaissances humaines," used the example of a "homunculus" on a cabbage leaf to show that the size of an observer will radically alter his interpretation of an event. Two months later Gaston Moch, in a similarly titled article, compared the perceptions of Crookes's

[95] Poincaré, *Science et méthode* (Paris: Ernest Flammarion, 1908), pp. 117-18; *The Foundations of Science*, p. 426.

[96] Poincaré, *Dernières Pensées* (Paris: Ernest Flammarion, 1913), pp. 63-70; trans. John W. Bolduc as *Mathematics and Science: Last Essays (Dernières Pensées)* (New York: Dover Publications, 1963), pp. 28-31.

[97] See Appendix A below for Poincaré's "law of relativity" for space, which he set forth in *La Science et l'hypothèse* of 1902 (p. 96).

homunculus specifically to those of a dweller in a two-dimensional world with no idea of a third dimension, or of a three-dimensional being who refused to admit the possibility of space of *n* dimensions.[98]

A 1903 *Revue Scientifique* article by Maurice Boucher, "La Relativité de l'espace euclidien," is typical of several such articles based on non-Euclidean geometry, which were generated by the French debate about the nature of geometrical axioms in the 1890s. Boucher's spatial relativism also extended to higher dimensions, and he was an important French advocate of the fourth dimension in this period. His *Essai sur l'hyperespace* of 1903 argues the very strong possibility of higher-dimensional space and explores the scientific and philosophic consequences of a fourth dimension. Throughout Boucher is supported by his firm belief in the relativity of knowledge: "Our senses, on the whole, give us only deformed images of real phenomena, some of which have long remained unknown, because none of our organs put us in direct contact with them. . . ."[99]

Boucher's work represents a more scientific hyperspace philosophy than that of Hinton. Although he drew upon Hinton's pioneering writings on the fourth dimension, Boucher himself had published articles in the *Revue Scientifique*, and his discussions of the possible relation of atomic theory, gravitation, and the ether to the fourth dimension have less a quality of pseudoscience than had Hinton's personal theories.[100] Boucher's twentieth-century hyperspace philosophy benefits from a science which, unlike strict positivism, was willing to admit the validity of the fourth dimension as a hypothetical concept.

Possible scientific applications of a fourth dimension of space had been discussed in the *Revue Scientifique* as early as 1891 in an article by René de Saussure. In "Sur une manière de considerer les phénomènes physiques et chimiques," de Saussure proposed that a "pression-chaleur" or "heat pressure" in the direction of the fourth dimension might be the underlying cause of the phenomena of light, heat, and electricity.[101] Serious scientific discussions of the fourth dimension such as that of de Saussure are

[98] See Crookes, "De la relativité des connaissances humaines," *Revue Scientifique*, 4th ser., VII (15 May 1897), 609-13; Moch, "Sur la relativité des connaissances humaines," *Revue Scientifique*, 4th ser., VIII (24 July 1897), 104-8.

[99] See Boucher, "La Relativité de l'espace euclidien," *Revue Scientifique*, 4th ser., XX (25 July 1903), 97-108; Boucher, *Essai sur l'hyperespace*, p. 64.

[100] Boucher quotes at one point from the 1898 French edition of Walter William Rouse Ball's *Mathematical Recreations and Problems of Past and Present Times* (1892), citing Ball's theory that gravitational attraction between particles might be explained if the ether were a four-dimensional homogeneous elastic body on which the particles rest (Boucher, *Essai*, pp. 154-56). An earlier chapter in Ball's book, "Hyperspace," summarizes contemporary popular views of the fourth dimension like those of Hinton, providing yet another possible source of this information for interested French readers. See Ball, *Recréations et problèmes mathématiques des temps anciens et modernes*, trans. J. Fitz-Patrick, from English 3rd ed. rev. (Paris: Hermann, 1898). The theories of Ball, as well as Hinton and others, are discussed in Alfred M. Bork, "The Fourth Dimension in Nineteenth-Century Physics," *Isis*, LV (1964), 326-38.

[101] See *Revue Scientifique*, 3rd ser., XLVII (9 May 1891), 585-88.

certain evidence that a move away from strict positivism was occurring in science toward the end of the nineteenth century.

By the time Maurice Boucher was writing in the early years of the twentieth century, science in general had developed beyond the stage when the label "positivist" could be applied to the majority of its practitioners. With the increase at the end of the nineteenth century in scientific research that could not be readily "sensed," the traditional positivist view was no longer tenable. Also, positivism as a philosophy of science had itself evolved, largely through the influence of Mach and his "pragmatic-economical" view of the human mind.[102] Mach's interpretation recognized that the scientist is not an objective recorder of the facts he observes; instead, he unconsciously orders the results of his observations in the ways that best suit his needs and interest and which are the simplest and most economical. While Mach maintained positivism's belief that observation and experiment are the proper activities of the scientist, he admitted that the scientist's results might well be influenced by the ways he personally tended to organize aspects of his own experience.

The changes in science toward the end of the nineteenth century which undermined direct observation as the universal scientific method soon encouraged a radical extension of the pragmatic-economical view. The central role in science, which had belonged to experience, was taken over by the creative activity of the scientist in his formulation of hypotheses. Of this more extreme approach to science as basically theory construction and, to a lesser extent, of Mach's pragmatic-economical view, Maurice Mandelbaum has written. "Neither held it to be within the scope of science (nor within the power of man) to say that one set of constructs more nearly approximated the characteristics of nature than did another: the test of the adequacy of a scientific theory lay wholly within the results which could be obtained by ordering past and future experiences in terms of that theory."[103] Poincaré's conventionalist views discussed in this chapter are a prime example of this last stage of nineteenth-century positivism, now transformed into creative theory building and sharing idealism's affirmation of individual freedom and creativity.[104]

Poincaré was to become a popular intellectual hero for the French during the early years of the twentieth century. His three major books on his philosophy of science,

[102] Mandelbaum, *History, Man and Reason*, pp. 16-19, 305-10.

[103] Ibid., p. 19.

[104] Ibid., p. 9. Because positivism had changed so substantially from its original tenets by the beginning of the twentieth century, the unqualified use of the term for this era can cause a great deal of confusion. Although historically Poincaré can be placed within the broad range of positivism's development, his contemporaries, such as Boucher and especially the French artistic avant-garde, did not associate him with positivism. In this and successive chapters, therefore, the term *positivist* is used only to describe the traditional, sense-oriented position of positivism, represented by Helmholtz.

published in 1902, 1904, and 1908, were widely read and served as major vehicles in the dispersion of knowledge about the new geometries. Non-Euclidean geometry was Poincaré's major concern, but his pronouncements on the fourth dimension had an equally great impact on a period in which ideas about higher dimensions became a widespread preoccupation.

Codification of the Fourth Dimension in the Early Twentieth Century: The 1909 Scientific American *Contest*

Popular literature on the new geometries during the first decade of the twentieth century focused primarily on "the fourth dimension" as opposed to non-Euclidean geometry. Between 1900 and 1910 the various notions about the fourth dimension advanced in the previous century had coalesced into a body of knowledge familiar to more and more of the public. This phenomenon is most apparent in the United States, where an abundance of popular magazines provided a natural arena for talk of this latest novelty. *The Popular Science Monthly* and *Science* had already begun featuring articles on the new geometries toward the end of the nineteenth century; the twentieth century saw the proliferation of articles on the subject in magazines such as *Harper's Weekly*, *McClure's*, and *Current Literature*.[105] The excitement in the United States culminated in 1909 when *Scientific American* sponsored an essay contest for "the best popular explanation of the Fourth Dimension," and entries were received from all over the world.[106]

The contest instructions had stated that the object was ". . . to set forth in an essay not longer than twenty-five hundred words the meaning of the term so that the ordinary lay reader could understand it."[107] The fourth dimension is interpreted by all of the entrants as a spatial phenomenon; time as the fourth dimension is not even mentioned. Most of the contestants are quite detached and noncommital about whether a fourth dimension exists, with only a few hyperspace philosophers among them advocating its reality.[108] Nevertheless, all of the essays borrow heavily from Hinton, who had

[105] A chronological listing of these American articles is provided in Appendix B, with full data in the Bibliography, sec. I, B, 4. Because Sommerville's concentration in his *Bibliography of Non-Euclidean Geometry* was upon more scholarly publications, few of these articles appear in his *Bibliography*. Instead, they are to be found in the *Readers' Guide to Periodical Literature* and other periodical indexes under "Fourth Dimension." In view of this fact, Sommerville's statement that by 1910, 1,832 works relating to the geometry of *n* dimensions had been published (p. viii) is all the more noteworthy, for the number would be far greater had the more popular articles been included.

[106] The editor of the published collection of these essays, Henry Parker Manning, mentions entries from Turkey, Austria, Holland, India, Australia, France, and Germany. See Manning, ed., *The Fourth Dimension Simply Explained*, p. 3.

[107] Ibid., p. 3.

[108] One of these hyperspace philosophers was the

formulated for the first time so many of the possible ways of understanding the fourth dimension.

The two-dimensional analogy that emerged in England, immortalized in Abbott's *Flatland*, plays a major role, as do Hinton's discussions of four-dimensional tesseracts bounded by cubes and of three-dimensional symmetry with objects being turned through a fourth dimension. Several authors also suggest the possibility of several three-dimensional worlds coexisting in a four-dimensional space. As H. G. Wells had discovered earlier, surprising effects would result if two of these separate worlds were brought into contact by a folding action like that with two corners of a piece of paper, representing analogous two-dimensional worlds in a greater three-dimensional space. Finally, Stringham's definitive work on the geometrical characteristics of the four-dimensional hypersolids fueled the more mathematical discussions, with contestants presenting tables listing the vertices, edges, and faces of the six hypersolids.

Most authors who mention Zöllner and his work with Slade portray the spiritualist connection as an unfortunate connotation taken on at times by the fourth dimension. The American contestants do not reflect the substantial contemporary interest in mystical or even Theosophical aspects of higher dimensions. Little impressed by Zöllner's pseudoscience, the essayists prefer to cite the "latest" scientific evidence for the possible existence of a fourth dimension. No reference to Einstein or Minkowski occurs, however, even though Minkowski had formulated the four-dimensional space-time continuum in 1908. Instead, "scientific" evidence consists of recent speculation on the nature of the ether and on atomic theory, along with certain of Hinton's personal ideas.

The mysterious ether had long permitted a good deal of hypothesizing for open-minded scientists, and it was a natural place for the fourth dimension to find its way into science. Often the actual scientific pronouncements of this era sound little less fantastic than Hinton's explanation of electricity as a four-dimensional vortex in the ether. Such is the case with Karl Pearson's "ether-squirts" theory, which suggested that the atom is a point at which ether is pouring into our space from a space of four dimensions.[109] Perhaps the most convincing scientific connection for the fourth di-

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American architect and designer Claude Bragdon, whose essay and subsequent publications are discussed in ch. 4.

[109] Carl A. Richmond, "How the Fourth Dimension May Be Studied," in *The Fourth Dimension Simply Explained*, ed. Manning, p. 90. On "ether squirts," see Walter William Rouse Ball, *Mathematical Recreations and Essays*, 5th ed. (London: Macmillan & Co., 1911), p. 465, an updated version of Ball's 1892 *Mathematical Recreations and Problems of Past and Present*, as well as Bork, "The Fourth Dimension in Nineteenth-Century Physics," pp. 334-35. Although none of the *Scientific American* essayists mentions the possible link between gravitation and the fourth dimension, Ball had done so in *Mathematical Recreations* in connection with the ether, and this association was often made in popular literature.

mension was in the field of chemistry. Chemists had long been puzzled by the problem of isomerism, where molecules of identical composition have different properties. It was now suggested that four-dimensional space would provide an extra point that could be positioned independently from the four points whose distances can be mutually independent in three-dimensional space.[110]

With the aid of hyperspace philosophy, Theosophy, fantasies like Abbott's *Flatland*, and the science fiction of Wells and others, the fourth dimension had become almost a household word by 1910. Non-Euclidean geometry never achieved such a widespread popularity, in part because it did not lend itself to such a variety of interpretations. Ranging from an ideal Platonic or Kantian reality—or even Heaven—to the answer to all of the problems puzzling contemporary science, the fourth dimension could be all things to all people. As a result, one of its most interesting aspects in early twentieth-century art is the variety of ways in which the fourth dimension was understood and then approached in visual terms by various artists in different countries.

[110] Graham Denby Fitch, "An Elucidation of the Fourth Dimension," in *The Fourth Dimension Simply Explained*, ed. Manning, p. 50.